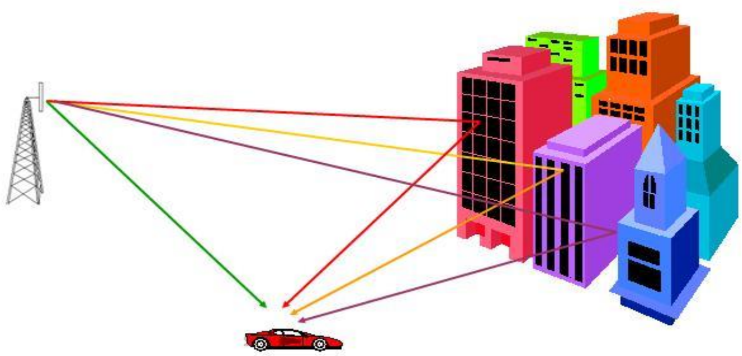
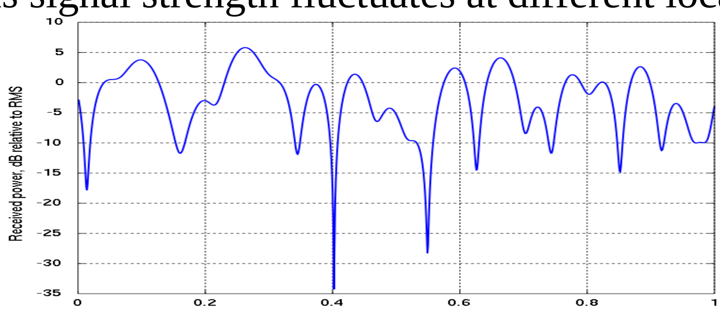
Chapter 3 Statistical Multipath Channel Models

Path loss and shadowing discussed in Chapter 2 are caused by large-scale characteristics of the environment. Therefore they are usually referred to as "large-scale" fading. (Some people prefer to use "large scale fading" only for shadowing.)

Radio wave reflection occurs at the earth surface as well as other objects. The location and the surface conditions of the objects are usually unknown to the transmitter and the receiver. The characteristic of the signals in mobile channel is a very complicated problem. This results in "small-scale" fading.

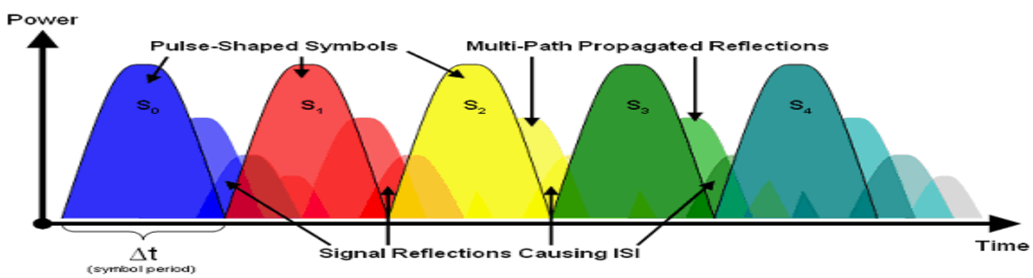


There are several effects of small-scale fading. The first is Rayleigh fading caused by multiple reflections of the transmitted signals arriving at the receiver at slightly different times. The reflections involved may be in-phase or out-of-phase to each other and may cause both amplitude and phase distortions, i.e., they may enhance each other or cancel each other. Thus signal strength fluctuates at different locations.



The second is that the carrier frequency of the received signal becomes randomly varying within a range determined by the so-called Doppler frequency.

The third occurs when the dimensions of the reflection objects are large and so the time difference among multiple reflections is large. Then the transmitted pulse can be “expanded” when it arrives at the receiver. When delay difference further increases, a receiver may receive multiple replicas of each transmitted pulse. In a digital system, such effect may cause inter-symbol interference (ISI).



In what follows, we will discuss the impact of relatively small-scale characteristics of the environment. The effect is sometimes referred to as small-scale fading.

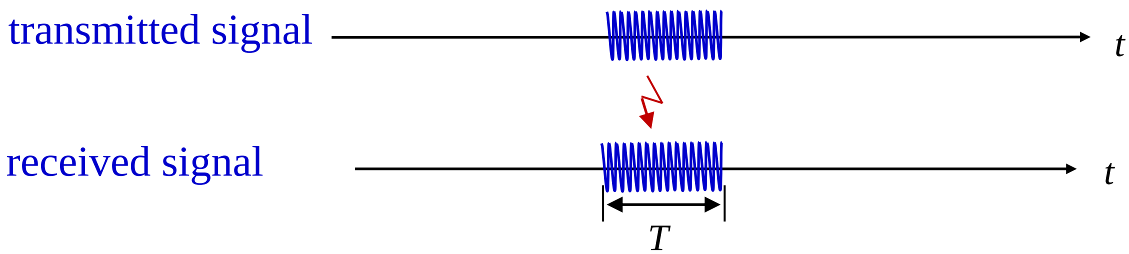
This Chapter includes the following parts.

3.1 Digital channel characterization
3.2 Rayleigh and Rician fading
3.3 Statistical channel modeling

Part 1. Digital channel characterization

Single-path channel

The following is an illustration of a simple single-path channel model.



Recall from the background discussions on digital communications. The transmitted signal is given by

$$s(t) = x_n^{\text{Re}} \cos(2\pi f_c t) - x_n^{\text{Im}} \sin(2\pi f_c t)\,. \quad (1)$$

For simplicity, we ignore the power control factor and assume that both x_n^{Re} and x_n^{Im} are not restricted in {-1, +1} as in QPSK. They can take any real values as in general quadrature amplitude modulation (QAM). The following notations are used:

$$x_n = x_n^{\text{Re}} + jx_n^{\text{Im}}, \quad (2a)$$

$$y_n = y_n^{\text{Re}} + jy_n^{\text{Im}}, \quad (2b)$$

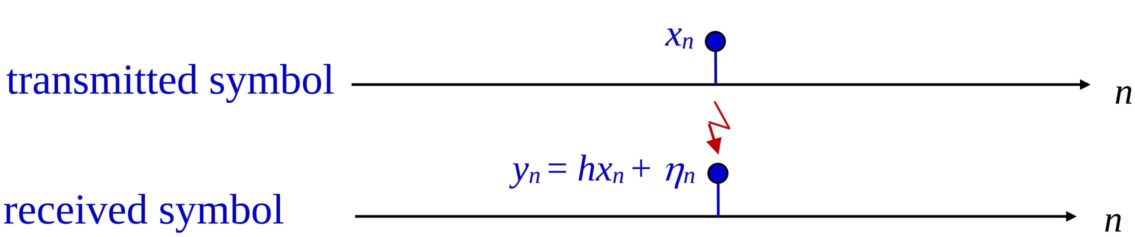
$$h = h^{\text{Re}} + jh^{\text{Im}}, \quad (2c)$$

$$\eta_n = \eta_n^{\text{Re}} + j\eta_n^{\text{Im}}. \quad (2d)$$

A single-path channel is characterized by a very simple expression:

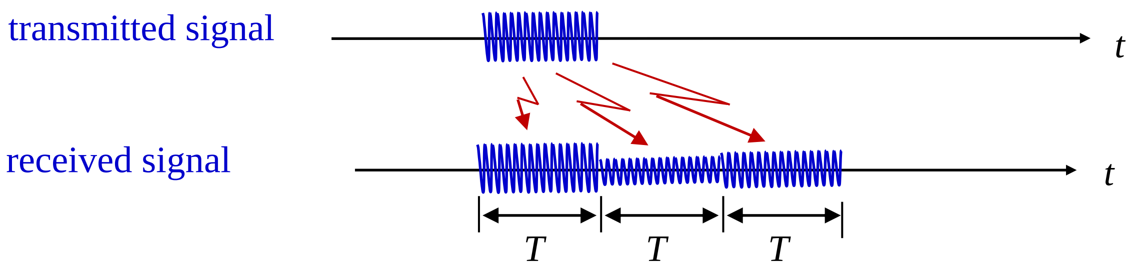
$$y_n = hx_n + \eta_n \quad (3)$$

Note that the above model includes a correlator receiver. Such a model is illustrated graphically below.



Multi-path channel

With reflections, multiple replicas may arrive at the receiver for each transmitted symbol. We first assume that each path delay is given by *kT*, where *k* is a non-negative integer.



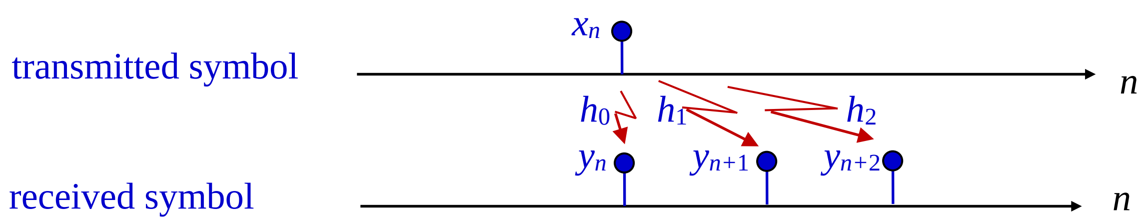
In this case, we can write

$$y_{n+k} = h_k x_n + \eta_{n+k} \tag{4a}$$

or equivalently

$$y_n = h_k x_{n-k} + \eta_n \quad . \tag{4b}$$

This is illustrated below.

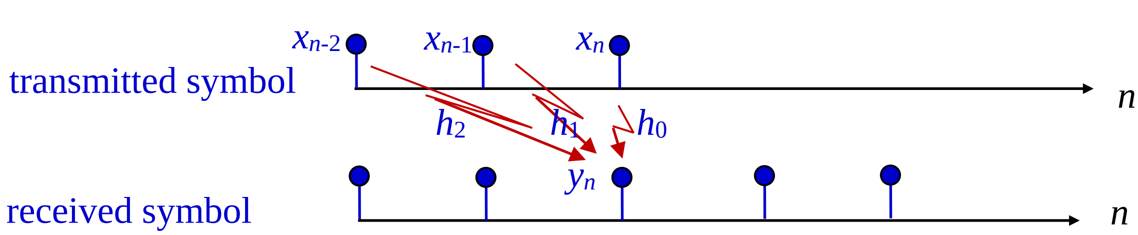


The above is for one transmitted symbol. We now consider multiple transmitted symbols

$$\ldots\ x_{n-2},\ x_{n-1},\ x_n,\ x_{n+1},\ x_{n+2},\ \ldots$$

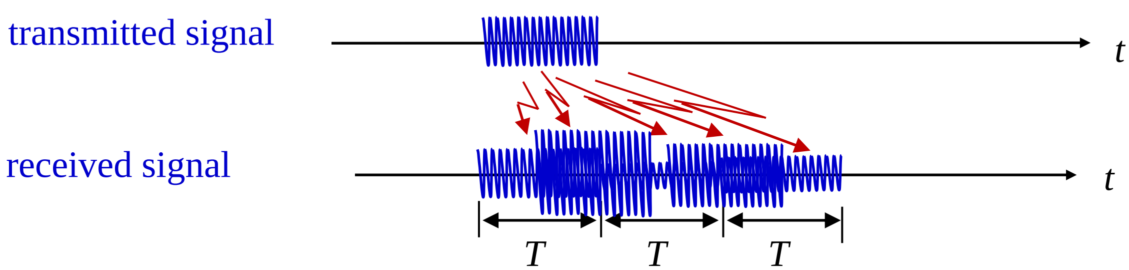
A received symbol is the sum of the responses of all the transmitted symbols. We can rewrite (4b) as

$$\begin{aligned} y_n &= h_0 x_n + h_1 x_{n-1} + h_2 x_{n-2} + \ldots + \eta_n \\ &= \sum_k h_k x_{n-k} + \eta_n . \end{aligned} \tag{5}$$

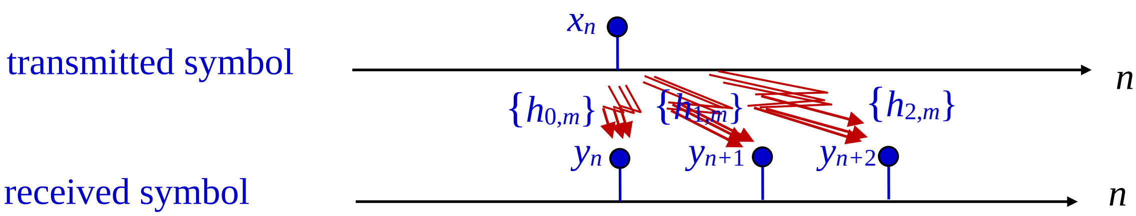


General multi-path channel

We now remove the restriction on delay. The delay for each path can be written as mT, where m is not necessarily an integer. The situation is illustrated below.



The following shows the situation after a correlation receiver.

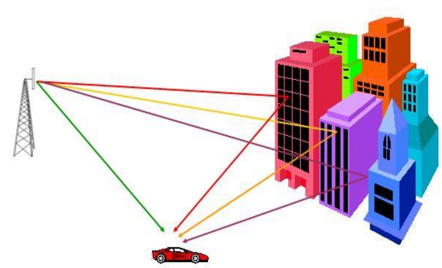


Each h_k includes the effect of several multipath reflections. It can be expressed as the sum of many terms, each representing a ray falling into period $(kT, (k+1)T)$:

$$h_k = h_k^{\mathrm{Re}} + jh_k^{\mathrm{Im}} = \sum_m h_{k,m}. \tag{6}$$

With multiple transmitted symbols, the channel model is the same as before:

$$y_n = \sum_k h_k x_{n-k} + \eta_n. \tag{7}$$



Notes:

(i) The above is for a time-invariant channel only.
(ii) Each h_k may include the effect of several reflections.
(iii) All variables can be complex. They represent the in-phase and quadrature signals carried by cosine and sine waveforms. The actual signals can be found by taking the real and imaginary parts.

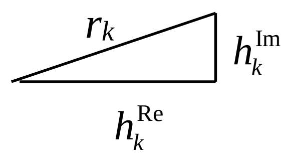
Part 2. Rayleigh and Rician fading

Rayleigh fading

Repeat (6):

$$h_k = h_k^{\mathrm{Re}} + jh_k^{\mathrm{Im}} = \sum_m h_{k,m}. \qquad (8)$$

From cnetral limit theorem, h_m^{Re} and h_m^{Im} are approximately Gaussian distributed. Let their mean = 0 and variances $=\sigma^2$. Define a new random variable

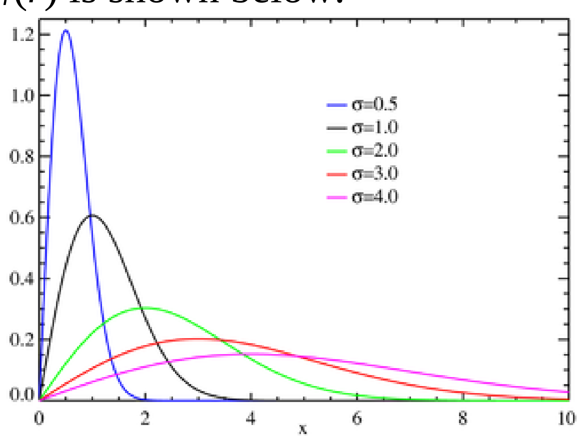


$$r_k = \sqrt{\left(h_k^{\mathrm{Re}}\right)^2 + \left(h_k^{\mathrm{Im}}\right)^2}. \qquad (9)$$

We treat r_k as a sample of a random variable r. When h_m^{Re} and h_m^{Im} are both Gaussian, r is Rayleigh distributed with PDF given by:

$$p_r(r) = \begin{cases} \dfrac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} & r \ge 0 \\ 0 & r < 0 \end{cases} \qquad (10)$$

The plot of $p_r(r)$ is shown below.



The mean value, mean power and variance of r are given by

$$E(r) = \sigma\sqrt{\pi/2}$$

$$E(r^2) = 2\sigma^2$$

$$Var(r) = E(r^2) - \left(E(r)\right)^2 = (2 - \frac{\pi}{2})\sigma^2$$

Exponential distribution

Let a and b be two independent Gaussian random variables with mean=0 and variance = σ^2. The random variable

$$z = (a^2+b^2) = r^2$$

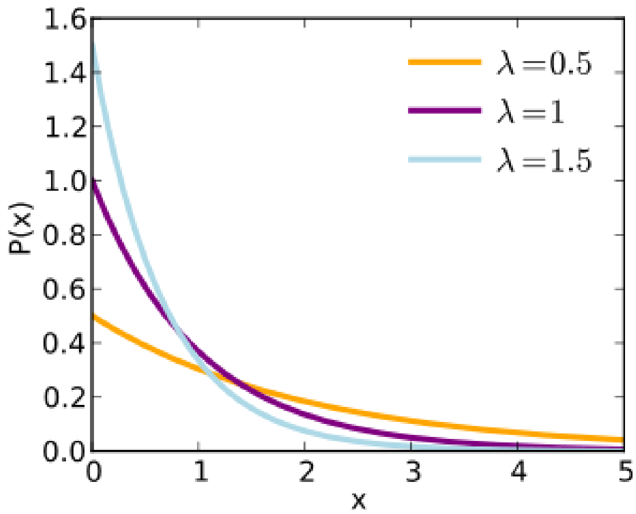
follows the exponential distribution. The PDF of z is given below.

$$p_z(z) = \begin{cases} \dfrac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}} & z \geq 0 \\ 0 & z < 0 \end{cases} \qquad (11a)$$

A commonly used alternative expression is (with $\lambda=1/(2\sigma^2)$)

$$p_z(z) = \begin{cases} \lambda e^{-\lambda z} & z \geq 0 \\ 0 & \text{otherwise.} \end{cases} \qquad (11b)$$

Its plot is given below.



The mean value and mean power of z is given by

$$E(z) = E(r^2) = 2\sigma^2$$

$$Var(z) = E(z^2) - (E(z))^2 = 4\sigma^4$$

Note: In many situations, we are interested in the average of a^2 and b^2:

$$p = z/2 = (a^2+b^2)/2$$

Questions:

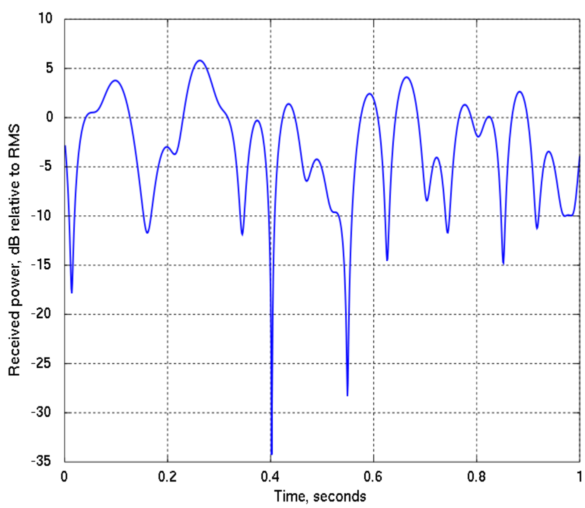
1) What is the physical meaning of p?
2) What is the distribution of p?
3) What is the mean and variance of p?

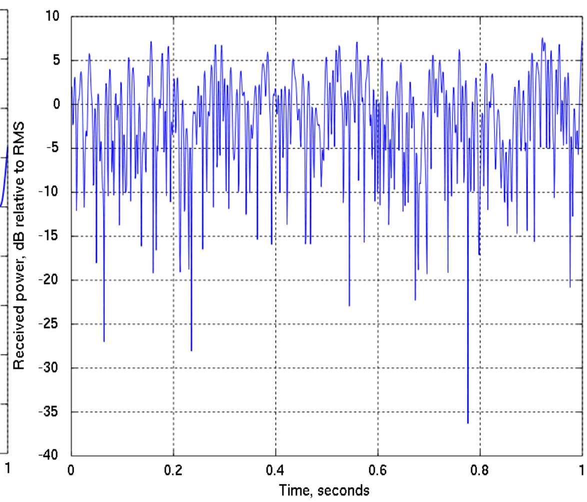
Illustration on Rayleigh fading

From the above, the distribution of the amplitude of a channel coefficient (also called a tap) is Rayleigh. The related fading is referred to as Rayleigh fading.

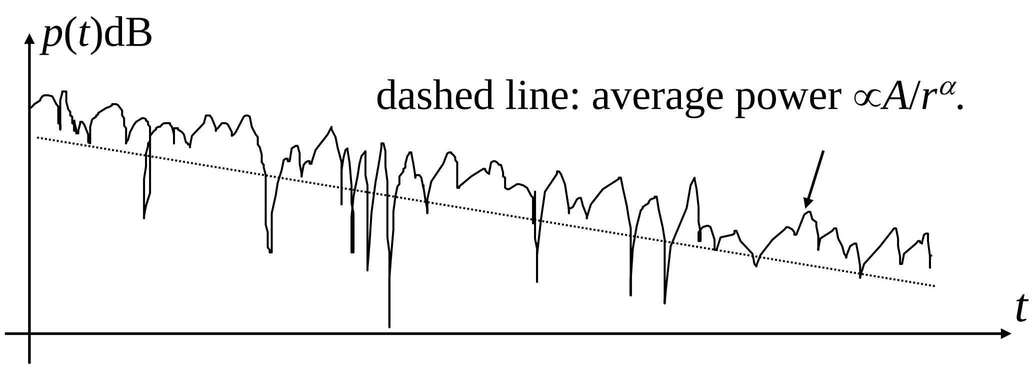
Traditionally, Rayleigh fading refers to both amplitude and power behaviors. Keep in mind that power actually follows exponential distribution for Rayleigh fading.

Two signals with the Rayleigh fading effect on moving terminals are illustrated in the figures below. They are one second snapshots of received power levels with a maximum Doppler shift of 10 Hz (left) and 100Hz (right), respectively.





The figure shown below further takes path loss into account



Rician fading

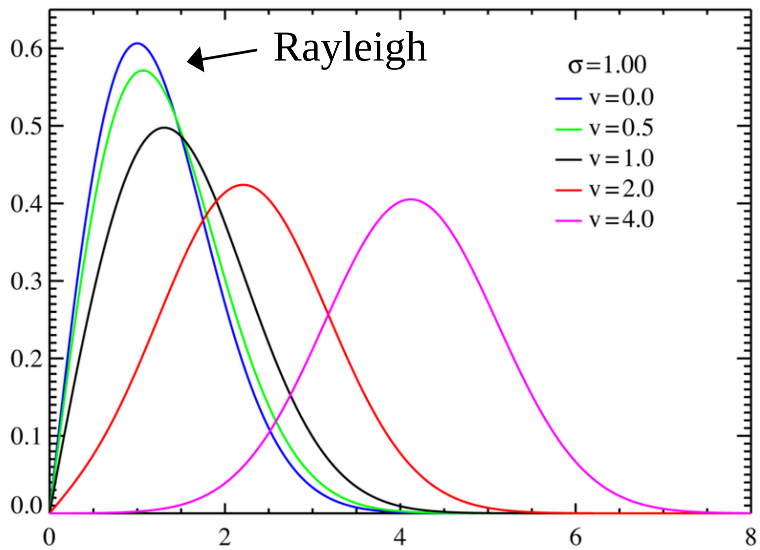
If there is a direct path, a dominant term exists. This means that there is a dominant term among different paths. In this case the law of large numbers cannot be applied.

It can be shown that the resultant signal envelope has a Rician distribution. The PDF of a Rician distributed variable is

$$p_r(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{(r^2+v^2)}{2\sigma^2}} \mathrm{I}_0\left(\frac{v\mathrm{r}}{\sigma^2}\right) & v \geq 0, r \geq 0 \\ 0 & r < 0 \end{cases}$$

where v^2 is the power in the line of sight (LOS) component and $\mathrm{I}_0(\cdot)$ is the modified Bessel function of 0th order.

The following is an illustration of Rayleigh and Rician distributions.



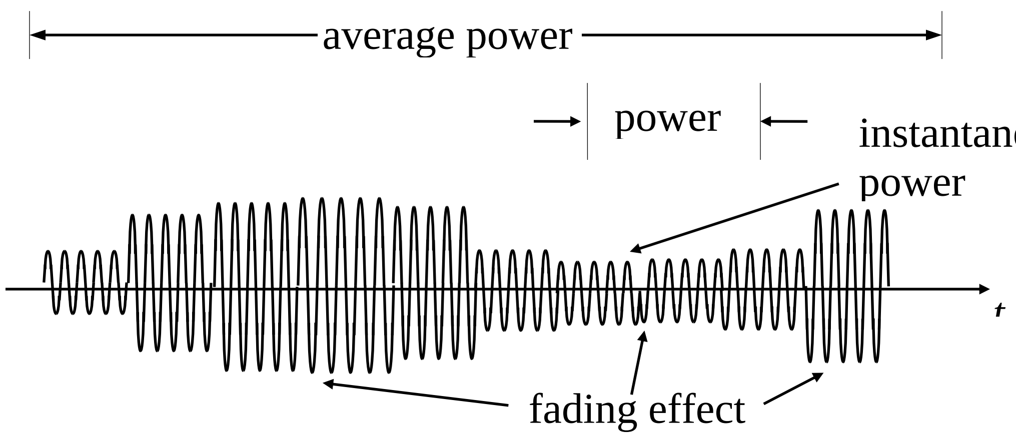
We can see that due to the existence of a dominating term, Rician distribution is more concentrated at a finite positive value. Clearly, Rayleigh fading is the limiting-case situation of Rician distribution when v=0.

Conventions on power and average power

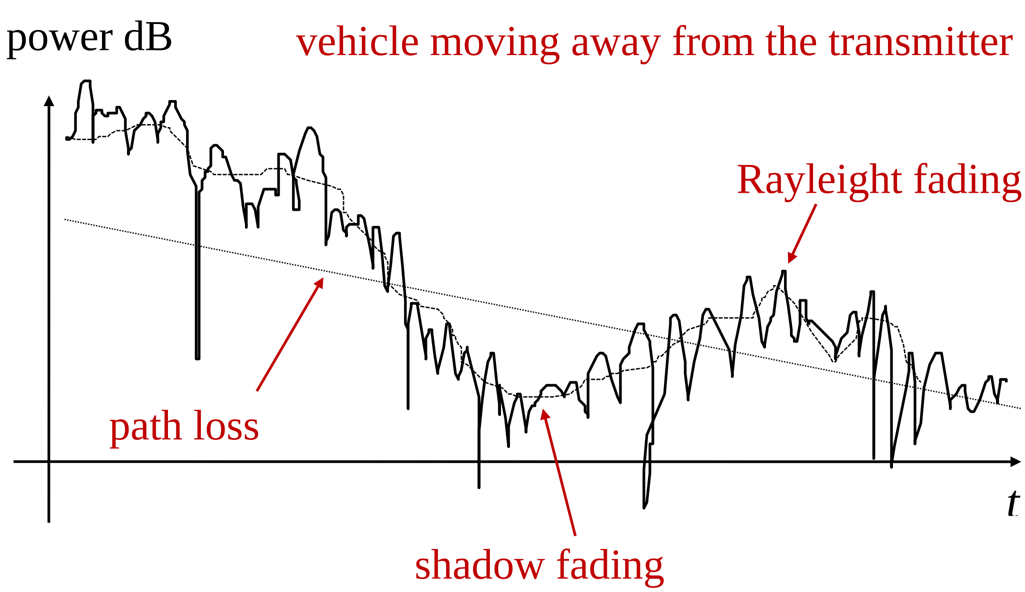
To avoid confusion, we will use the following convention about power:

- Instantaneous power = $|s(t)|^2$.
- "Power" refers the local time mean of the instantaneous power, i.e., averaged over a period of cosine and sine functions. (For example, the average power of a cosine function is 0.5.) Notice that the average power is still a random variable, subject to fast or slow fading.
- Long term average power = $E[|s(t)|^2]$. This is averaged over fading effects. Here long term average fading can be further divided into large-scale and small-scale fading cases.

The above is illustrated by the following graph.



Usually we will not discuss instantaneous power. We will mostly discuss power or average power.



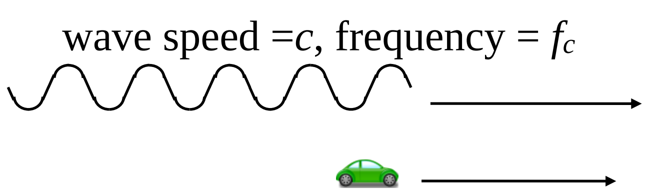
Part 3. Statistical channel modeling

Doppler effect

We now discuss time varying channels related to, e.g., moving vehicles. We will focus on a narrowband transmitted signal $\cos(2\pi ft)$. When a vehicle is moving, its speed will affect the frequency of the received or the transmitted signal, which we call Doppler effect.

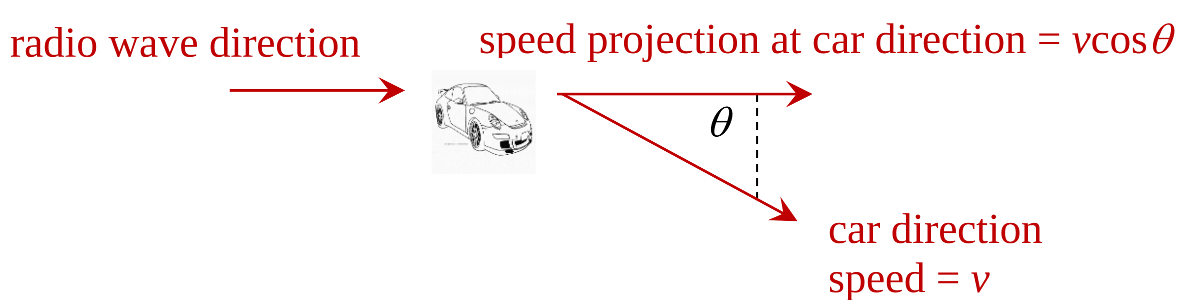
Consider an EM wave with carrier frequency f_c. Assume that a car is traveling in the same direction of the wave. If the speed of the car = 0, the wave received in the car apparently has frequency f_c. On the other hand, if the car is traveling at the speed of v=c (speed of light), then the wave received in the car should have an apparent frequency = 0. (Why?) For any speed in between, the frequency of the wave relative to the car is given by,

$$f = (1 - \frac{v}{c}) f_c$$



If the directions in which the car and the wave travel have an angle, then

$$f = (1 - \frac{v\cos\theta}{c}) f_c \, .$$



Doppler frequency

From the above, the Doppler effect introduces an distortion term in the frequency of the received signal measured by the Doppler shift

$$f_c - f = \Delta f = \frac{v f_c}{c}\cos\theta . \qquad (12)$$

The maximum value of Δf (over all possible Δf) is the so-called Doppler frequency given below:

$$f_D = \frac{v f_c}{c} = \frac{v}{\lambda} \quad . \qquad (13)$$

The Doppler shift causes a carrier frequency shift in the received signal. For example, for a vehicle with v=72km/h=20m/s and f =1000MHz (λ=0.3m), the maximum Doppler shift is f_D=20/0.3=66.7Hz.

Doppler spectrum

With the Doppler effect, different rays of the received signal may have different frequencies, implying that h_k will now change with time. Such changing is random but can be characterized by its statistical power spectrum. Aassume that the arrival angle of each ray is uniform distributed. The statistical power spectrum of the received signal is given by the so-called Doppler power spectrum below.

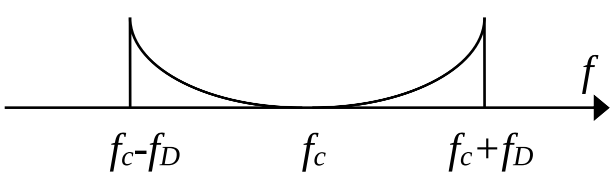
(See Jakes' model https://en.wikipedia.org/wiki/Rayleigh_fading.)

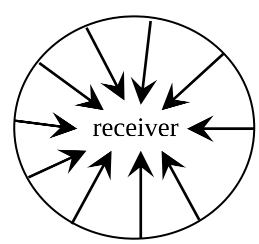
$$S_r(f) = \frac{P_r}{2\pi f_D}\frac{1}{\sqrt{1-\left(\left|f-f_c\right|/f_D\right)^2}}$$

With normalized, we define a Doppler power spectral density (PSD) as

$$S(f) = S_r(f) / \int_{-\infty}^{\infty} S_r(f)\,df$$

Doppler PSD





Physical meaning of Doppler spectrum

We may roughly interpreted a PSD as follows. We transmit a single-tone signal at frequency f_c. Then a PSD gives the probability density that the received signal for a particular path has frequency f. As seen from the PSD above, f has high probability at $f_c \pm f_D$.

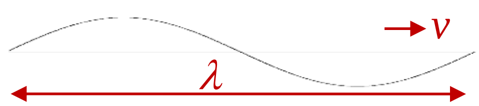
Channel coherent time

The channel coherent time, denoted by T_c, of a time-varying channel is the duration that channel is approximately unchanged.

An approximate expression for T_c is

$$T_c \approx k / f_D = k\lambda / v ,$$

where f_D is the Doppler frequency and k is a constant. A typical value of k is 1. (Sometimes k=0.5 is also used). Here λ/v is the time to travel one wavelength at speed of light. Thus the above says that the signals separated by more than one wavelength apart are approximately uncorrelated.

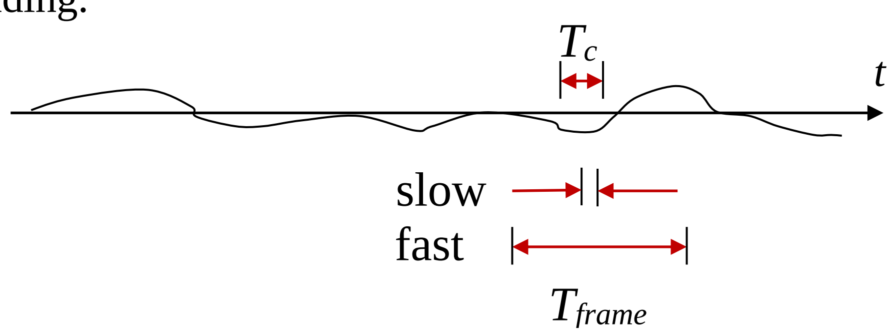


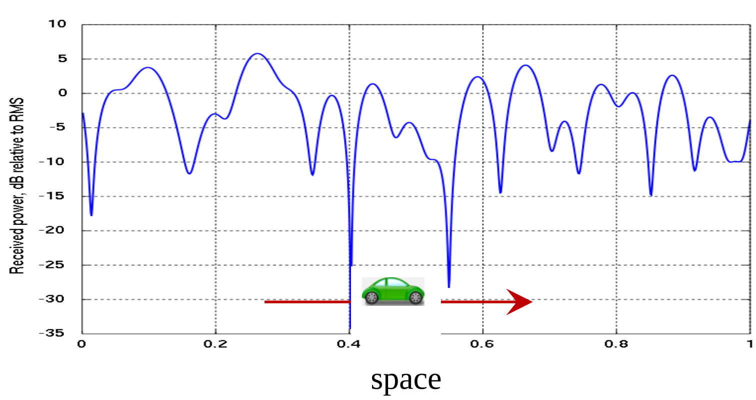
Slow and fast fading

Let the length of a signal frame be T_{frame}. We say that the system is experiencing slow fading if

$$T_{frame} \ll T_c .$$

On the other hand, if $T_{frame} \gg T_c$, we say that the system is experiencing fast fading.



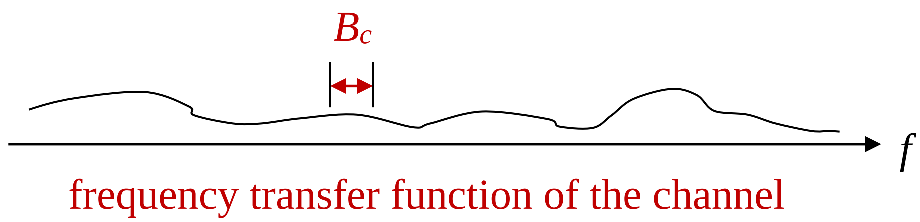


Channel coherent bandwidth

Similar to the channel coherent time defined earlier, the coherent bandwidth, denoted by B_c, is the frequency range that channel can be approximately regarded as unchanged. A commonly used rule is

$$B_c \approx k / \sigma_{Tm},$$

where σ_{Tm} is the rms delay spread and k is a constant. (We will discuss rms delay spread later.) A typical value of k is 1. (Sometimes k=0.5 is also used).



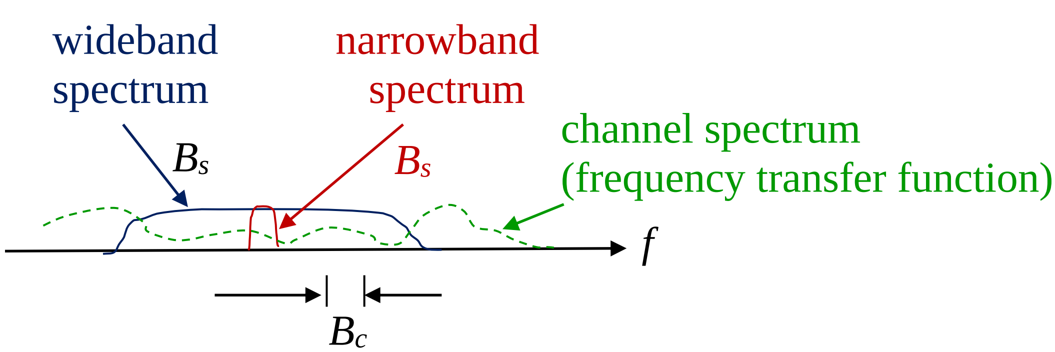
Narrowband and wideband systems

Let the bandwidth of the signal be B_s. We say that the system is narrowband if

$$B_s << B_c .$$

Otherwise, we say that the system is wideband.

Note that "wideband" or "narrowband" is relative to the channel coherent bandwidth. It is not determined by signal bandwidth alone.

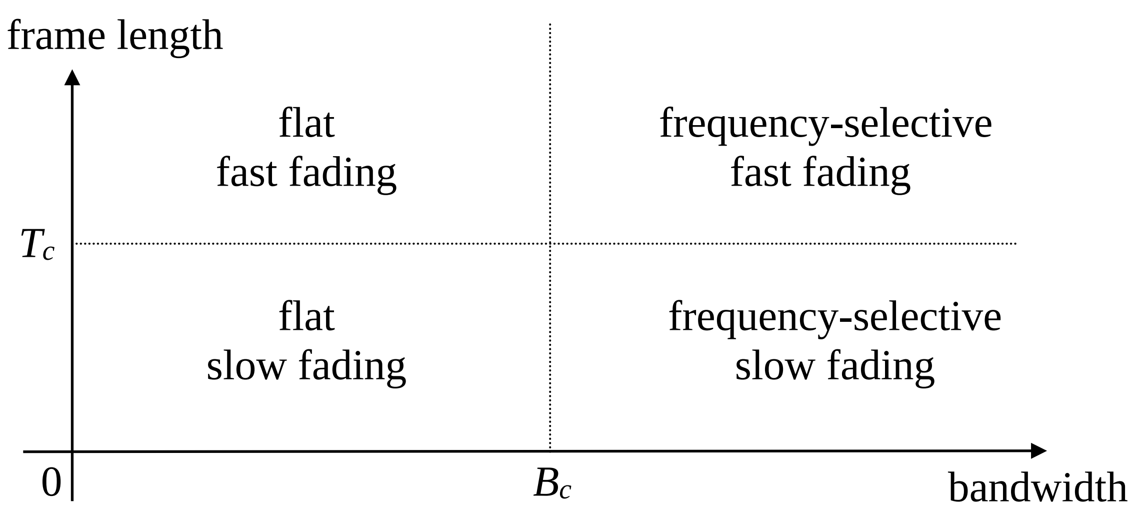


Notes:

- A narrowband system is experiencing flat fading. In this case, the received power spectrum is roughly flat.
- A wideband system is experiencing frequency-selective fading channel. In this case, the received power spectrum varies at different frequency range.
- "Fast" or "slow" is determined by frame length, and Doppler spread. (Recall that $T_c \approx k / f_D$.) (The latter is in turn determined by vehicle speed and carrier frequency).
- "Flat" or "frequency selective" are determined by system bandwidth and delay spread. (Recall that $B_c \approx k / \sigma_{Tm}$.)

Summary on different types of fadings

The following figure summarizes different types of fading channels.



Notes: The terms "slow" or "fast", however, are sometimes used with other meanings. Slow fading may mean "large-scale" fading including shadow fading and path-loss caused by building or large objects. Fast fading may mean small-scal fading, i.e., Rayleigh fading, caused by multiple reflections. This is somewhat confusing and you should be careful about the usage.

Power delay profile (PDP)

In a time varying channel the impulse response changes with both time and position. It can be regarded as a random function. Power delay profile (PDP), denoted by $f_{PDP}(\tau)$, is commonly used to characterize the power dispersion of signal in the time domain in a certain transmission environment.

Roughly speaking, a PDP $f_{PDP}(\tau)$ gives the average received power at time $t+\tau$ for an impulse transmitted at time t. The expectation is over time and position.

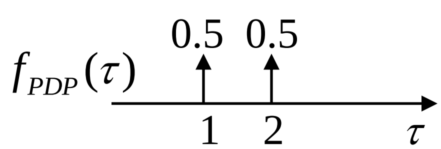
The underlying assumption of the above definition is that the channel is wide-sense stationary and ergodic, which roughly mean the following.

- The statistical behavior of a wide-sense stationary channel is unchanged over all time.
- The statistical behavior of an ergodic channel is unchanged over all positions and all tries.

Keep in mind that the above are only assumptions.

PDP after correlation receiver

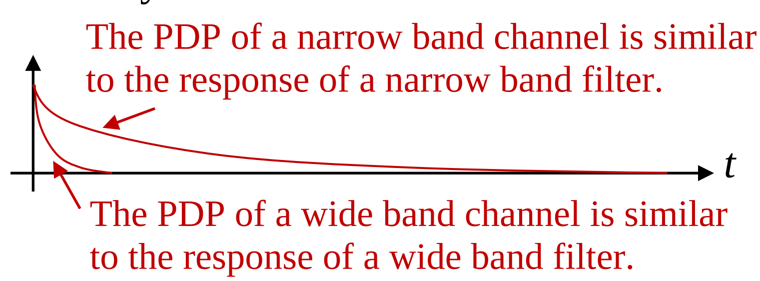
A PDP after a correlation receiver becomes discrete. For example, consider $f_{PDP}(\tau) = 0.5 \cdot \delta(\tau - 1) + 0.5 \cdot \delta(\tau - 2)$.



This means that in average the received power is 0.5 at both τ=1 and τ=2.

Connection of PDP and coherent bandwidth

We can treat the channel as a filter. If an impulse response decays slowly, than the bandwidth a filter is a relatively narrow. Otherwise, the bandwidth is relatively wide.



Average delay and r.m.s. delay spreads

Given a $f_{PDP}(\tau)$, we defined a normalized PDP as:

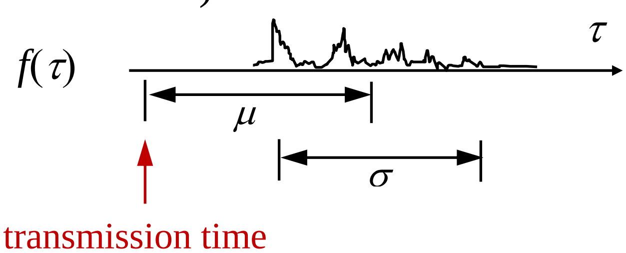
$$f(\tau) = \frac{f_{PDP}(\tau)}{\int_0^\infty f_{PDP}(\tau)d\tau}.$$

We can then define average delay and r.m.s. delay spread as

average delay: $\mu = \int_0^\infty \tau f(\tau)d\tau$

r.m.s. delay spread: $\sigma = \sqrt{\int_0^\infty (\tau-\mu)^2 f(\tau)d\tau}$

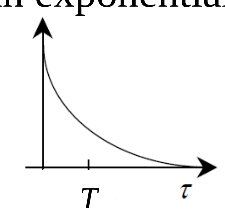
Here μ is a measure of transmission delay with respect to transmission time, while σ is a measure of the range of received power. (We can view μ as mean value and σ as standard deviation.)



Continuous exponential PDP

A common normalized PDP is given by an exponential distribution.

$$f(\tau) = \frac{1}{T}e^{-\frac{\tau}{T}}$$

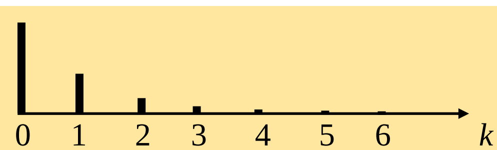


It can be shown that $\mu = \sigma = T$ in this case. Other PDPs may not have such a property.

Discrete exponential PDP

A common normalized discrete PDP is also given by a discrete exponential distribution.

$$f(k) = \left(1-e^{-\frac{1}{T}}\right)e^{-\frac{k}{T}}$$



Again, we can view $f(k)$ as a distribution.

Physical meaning of normalized PDP

A normalized PDP can be seen as a distribution. Roughly speaking, it has the following meaning: We transmit an impulse signal at time t. Then

- a continuous PDP gives the probability density that the received impulse on a particular path is at time $t+\tau$; and
- a discrete PDP gives the probability that the received signal on a particular path is at time $t+kT$, where T is the symbol duration.

Channel simulation

The field experiment of a wireless system is costly. Computer simulation is a more efficient way for assessing system performance. Generating channel coefficients is a key step in simulation.

Combining the discussions above, we model the channel coefficients of a multi-path, time-varying channel with Doppler effect as follows. Return to (6)

$$h_k = \sum_m h_{k,m}.$$

We assume the channel coefficients are functions of t and write them as $\{h_{k,m}(t)\}$. Each $h_{k,m}(t)$ corresponds to a transmitted signal at time t and can be expressed as:

$$h_{k,m}(t) = A\alpha_{k,m} e^{-j2\pi\left(\Delta f_{k,m} t + \phi_{k,m}\right)}.$$

Here $\alpha_{k,m}$, $\Delta f_{k,m}$ and $\phi_{k,m}$ represent, respectively, amplitude, frequency shift and initial phase caused by a reflection. They are all random variables modelled as follows.

i) Randomly draw $\alpha_{k,m}$ using a discrete PDP $f(k)$ (such as an exponential distribution). Specifically, $\alpha_{k,m}=1$ with probability $f(k)$ and $\alpha_{k,m}=0$ with probability $1-f(k)$.

ii) Randomly draw $\Delta f_{k,m}$ based on a Doppler PDF determined by vehicle speed.

iii) Randomly draw $\phi_{k,m}$ using a uniform distribution.

Note that m is an index of random samples. If a sufficient number of samples are taken, the summation in (6) results in Rayleigh fading.

Chapter 3 summary

1) Equivalent discrete channel model

$$y_n = \sum_k h_k x_{n-k} + \eta_n \,.$$

2) Rayleigh and Rician distributions. In particular, the Rayleigh distribution is define by

$$p_r(r) = \begin{cases} \dfrac{r}{\sigma^2} \cdot e^{-\frac{r^2}{2\sigma^2}} & r \geq 0 \\ 0 & r \geq 0 \end{cases}$$

An exponential distribution is defined by

$$p_z(z) = \begin{cases} \dfrac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

3) The Doppler frequency is given by

$$f_D = \frac{v}{c} f_c$$

The related Doppler PSD is given by

$$p_f(f) = \frac{P_r}{2\pi f_D} \frac{1}{\sqrt{1-(|f - f_c| / f_D)^2}}$$

4) Channel coherent time: $T_c \approx k / f_D = k\lambda / v$. Typically, *k*=1.

Channel coherent bandwidth: $B_c \approx k / \sigma$. Typically, *k*=1.

5) Different fading types:

- flat fast fading
- frequency-selective fast fading
- flat slow fading
- frequency-selective slow fading

6) PDP:

Continuous: $f_{PDP}(\tau) = (1/T)e^{-\tau/T}$

Discrete: $f_{PDP}(k) = (1-e^{-1/T})e^{-k/T}$