Chapter 4

Cellular Capacity

Outline

Introduction
4.1 System architecture
4.2 Interference problem
4.3 Spectrum efficiency
4.4 Multiple access techniques

Introduction

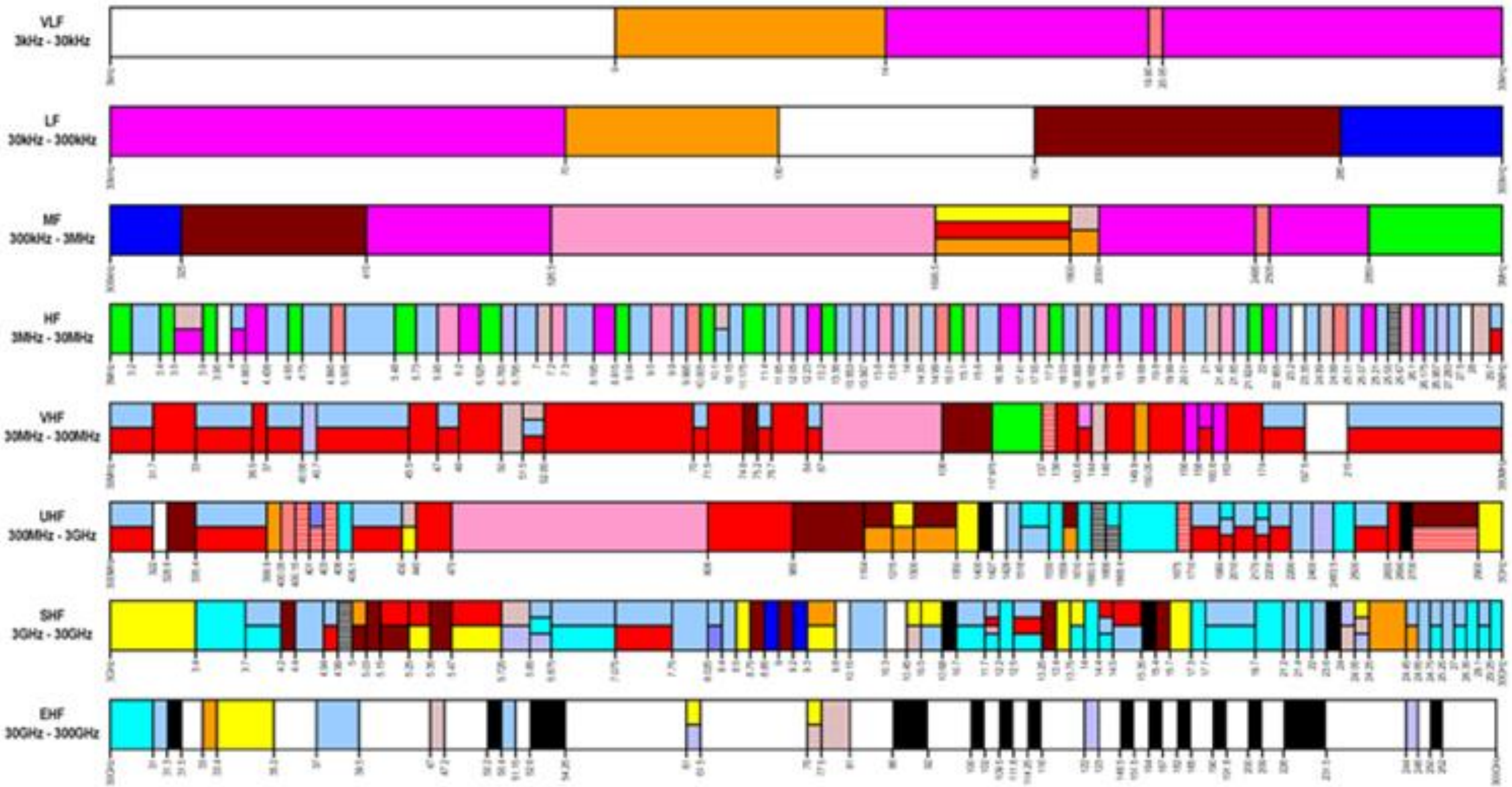
Introduction

Free space frequency spectrum is a very scarce resource. Therefore its use is strictly controlled by the regulatory authorities in most countries. One way to efficiently use the available spectrum is the cellular technique, which makes the modern public mobile networks possible.

The basic concept is “frequency reuse by cellular technique” which was invented in the middle of the last century by the Bell Labs. If a channel with a certain frequency covers an area of radius *R*, then the same frequency can be reused to cover another area. Each one of the areas is known as a cell. Cells using the same frequency are positioned sufficiently apart from each other so that co-channel interference can be within tolerant limits due to the propagation attenuation. This philosophy is used in 1G and 2G systems. For 3G and 4G systems, more sophisticated resource (such as frequency spectrum) allocation techniques are used, as will be discussed later.

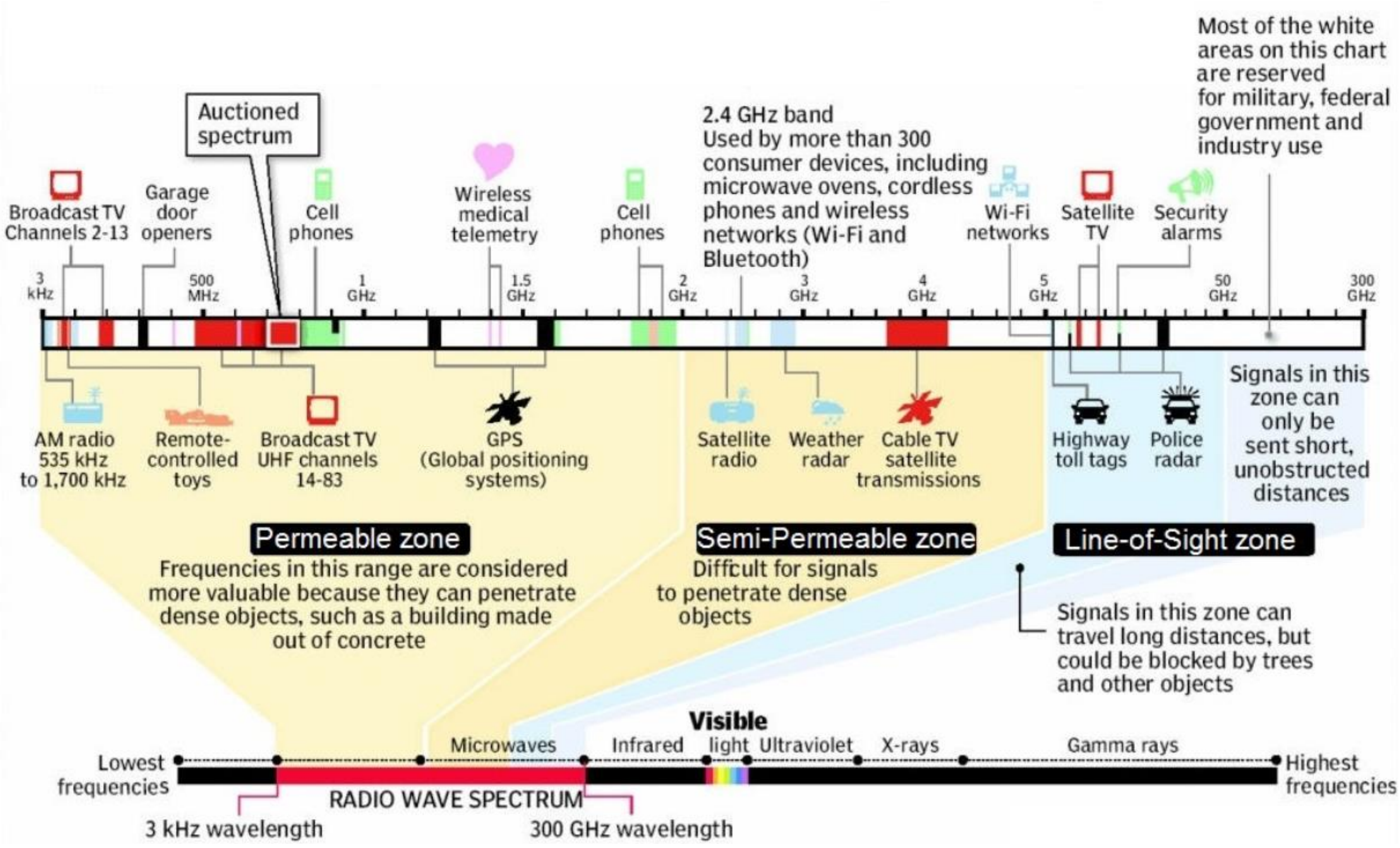
Radio Spectrum Allocation

The following is the radio spectrum allocation for different applications, such as cellular phone, police, emergency, airport, maritime, TV, …, in Hong Kong.



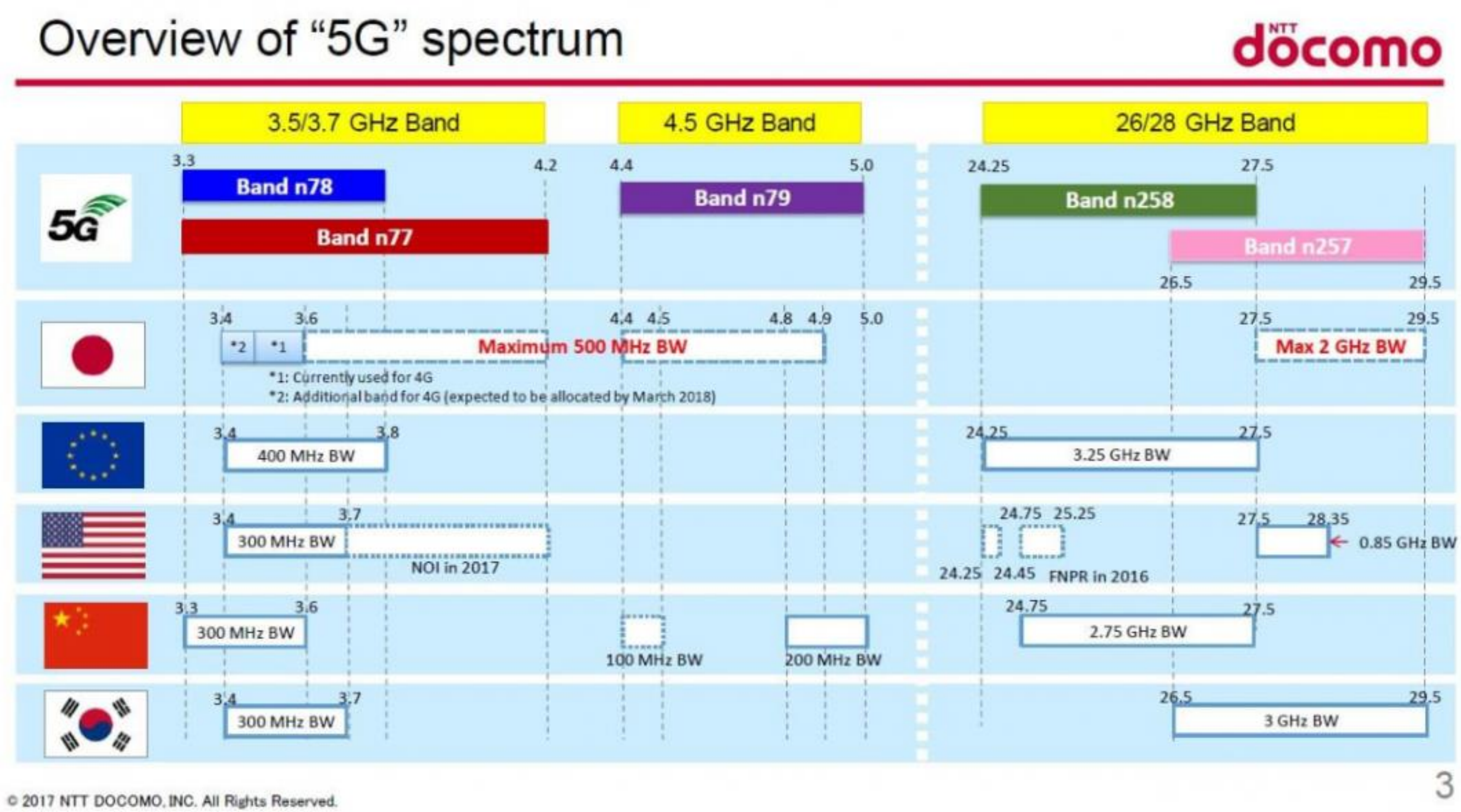
1G – 4G Spectrum Allocation

The spectrums for 1G to 4G are allocated to relatively low frequency range below 3GHz.

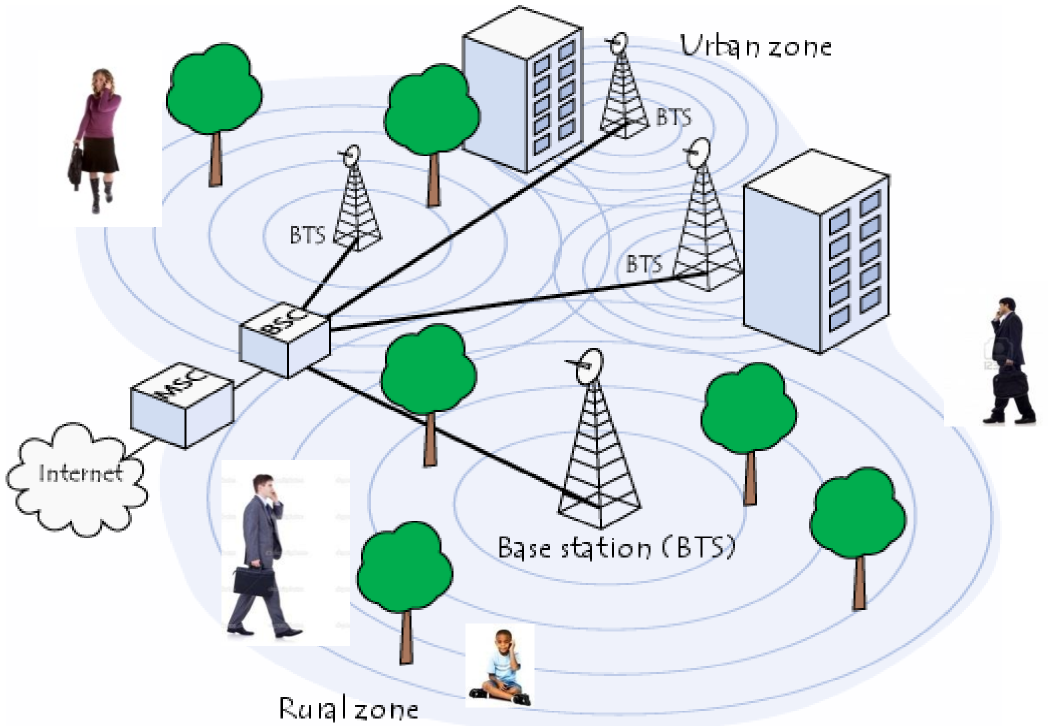


5G Spectrum Allocation

The spectrums for 5G are in higher frequency range.



Introduction



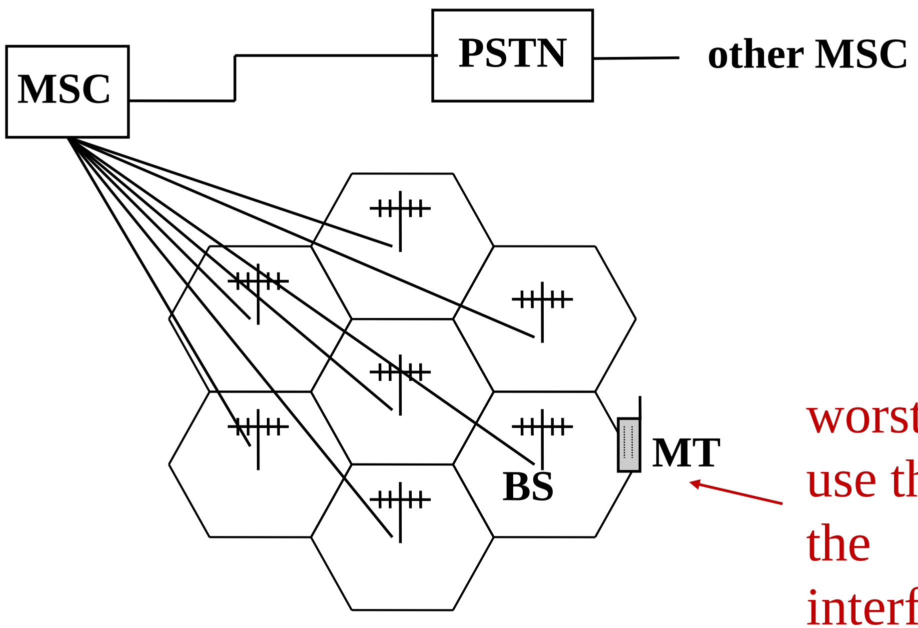
Chapter outline:

Part 1. System architecture
Part 2. Interference problem
Part 3. Spectrum efficiency
Part 4. Multiple access techniques.

4.1 System architecture

Cellular concepts

A cell is typically an area of hexagon shape. The basic system architecture is shown below.

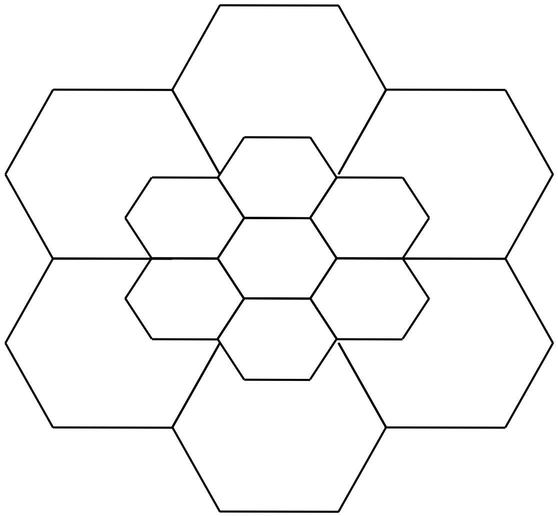


worst case position:
use the most power,
the most prone to interference,
crossing border

BS: Base Station
MSC: Mobile Switching Center
PSTN: Public Switching Telephone Network
MT: Mobile Terminal

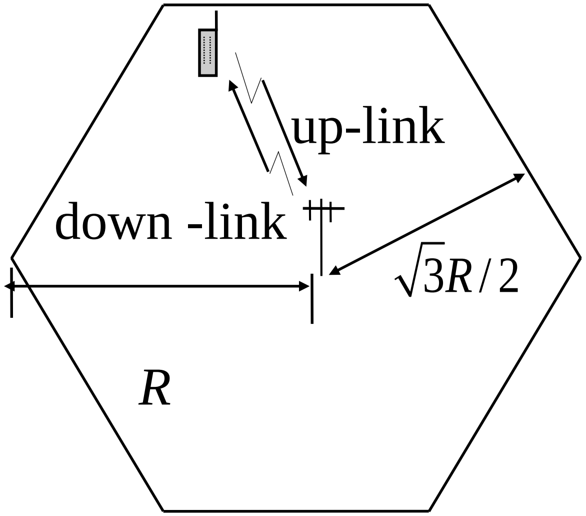
Cellular concepts

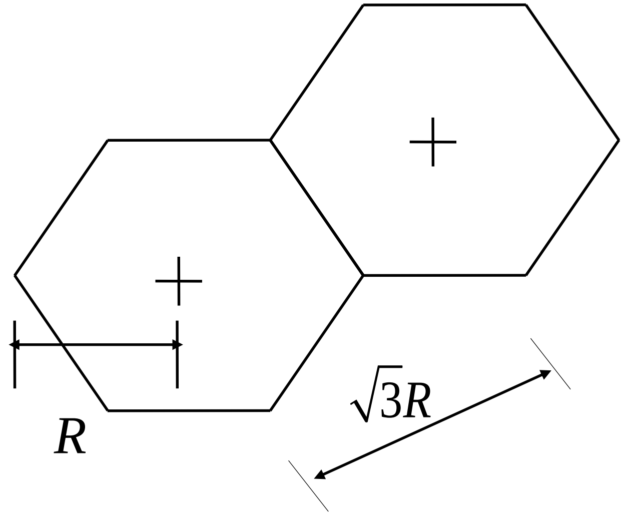
A practical cellular system may have irregular cell pattern to cope with non-uniform user distribution. The smaller the cell size, the more users density supported. For convenience, we will only discuss uniform hexagon cell planning.



Cell and base station

An ideal cell is typically represented in a hexagon shown below. (In practice, this is certainly just an approximation.) The reason to use a hexagon instead of a square or a circle is that a hexagon is the most close to a circle among all the shapes that can be replicated to cover an area without gap.





Cell and base station

The BS is normally located at the cell center. In practice, we will say that the distance from the BS to the cell border is R. This is only approximately true since R is actually the distance from the BS to the six corners of the border (the worst case).

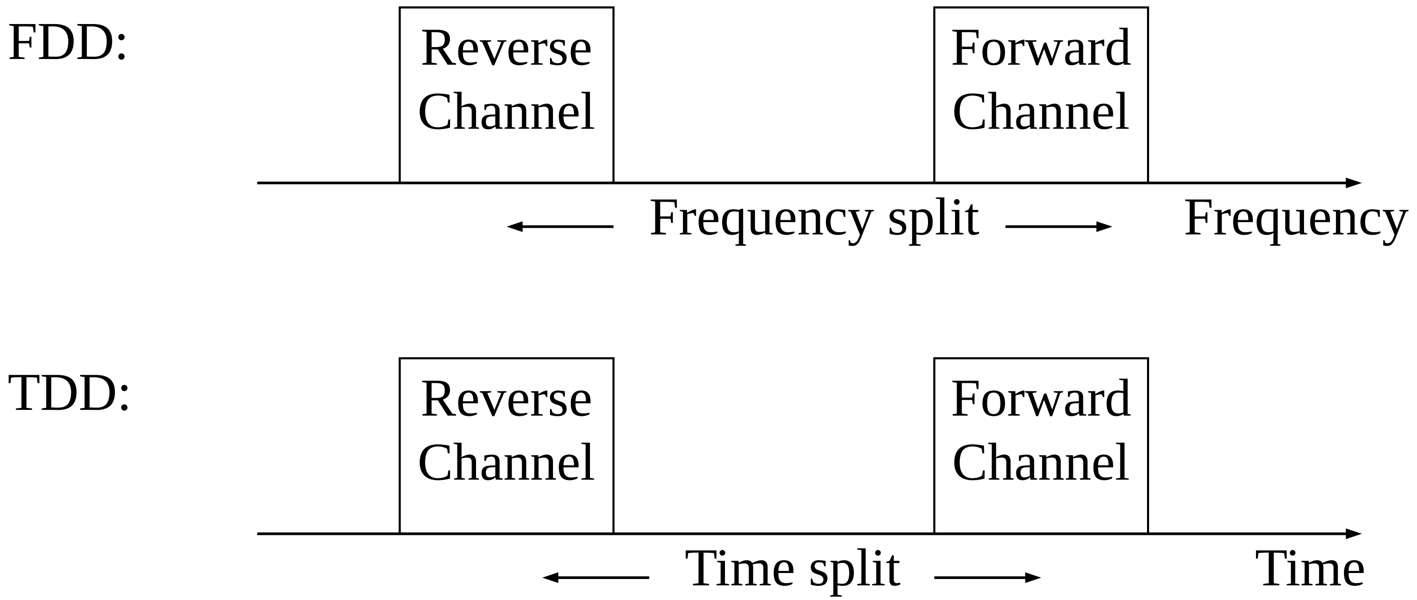
The distance between two adjacent cell centers is $\sqrt{3}R$.

There are two channels between the BS and a MT, called down-link (BS to MT, alternatively called forward link in US) and up-link (MT to BS, alternatively called reverse link in US), respectively.

If up and down links occupy different spectrum, the system is said "frequency division duplexing" (FDD).

If up and down links occupy the same spectrum but different time slots, the system is said "time division duplexing" (TDD).

Cell and base station



Distances between two cell centers

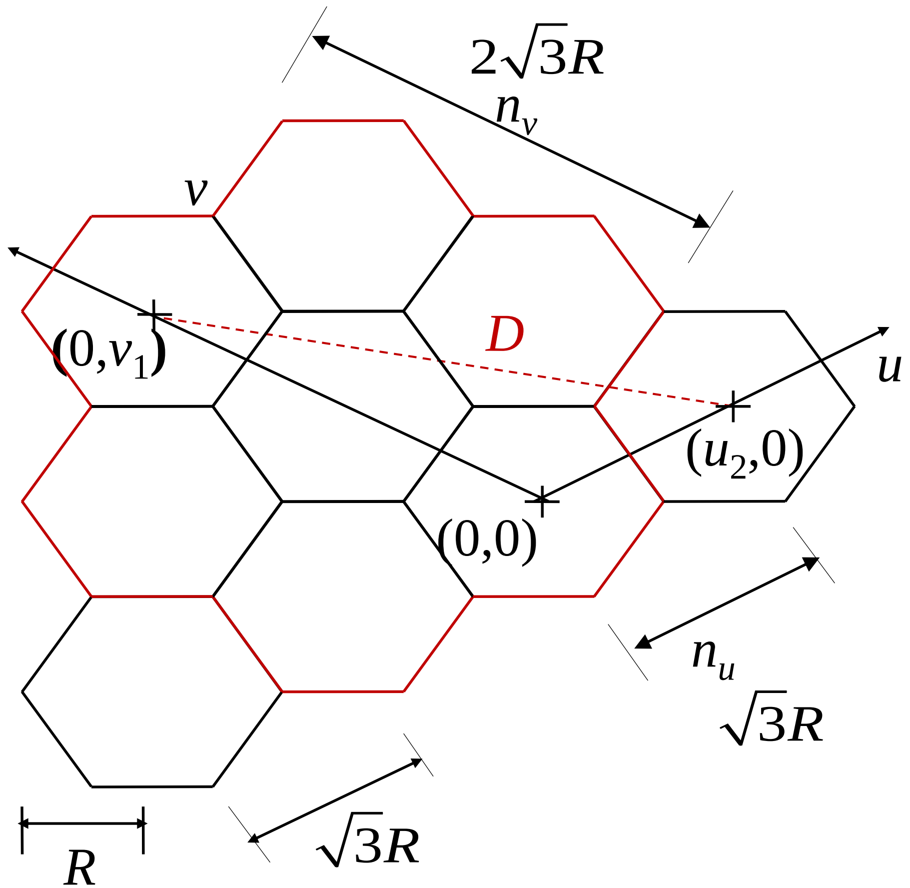
Given two cell centers, we can draw two lines with an angle of $\theta = 2\pi/3$ as shown below. We establish the origin (0, 0) as the cross point of the two lines. Let the two cell centers are labeled as $(0, v_1)$ and $(u_2, 0)$.

The distance between $(0, v_1)$ and $(u_2, 0)$ can be calculated as follows:

$$
\begin{aligned}
D^2 &= v_1^2 + u_2^2 - 2v_1u_2\cos(2\pi/3) \\
&= v_1^2 + u_2^2 + v_1u_2 \\
&= (\mathrm{n}_\mathrm{v}^2 + n_u^2 + n_v n_u)3R^2 \qquad (4.1)
\end{aligned}
$$

where n_v and n_u are the numbers of cell centers from (0, 0) to $(0, v_1)$ and $(u_2, 0)$, respectively. For the example shown below, $n_u = 1$ and $n_v = 2$.

Distances between two cell centers



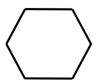
Cells cluster and frequency reuse pattern

A cluster is a group of contiguous cells assigned with different frequencies. This gives the basic frequency reuse pattern. Such pattern is repeated to cover the entire system area.

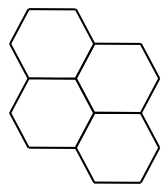
The following are commonly used clusters. We will refer to $N \in \{1, 3, 4, 7, 12\ldots\}$ as a regular reuse number. A cluster can be replicated to cover a continuous area. A cluster with a larger N can achieve larger distance between the co-cell (cells using the same frequency) than one with a smaller N.

Cells with the same index are assigned with the same set of frequency channels so they may interfere with each other. Cells with different indices are assigned with different and, strictly speaking, non-overlapping sets of frequencies, so they will not interfere with each other.

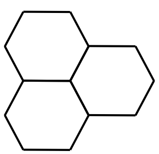
Cells cluster and frequency reuse pattern



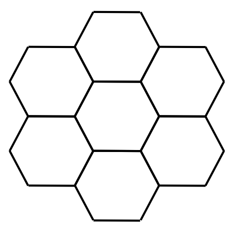
1-cell cluster



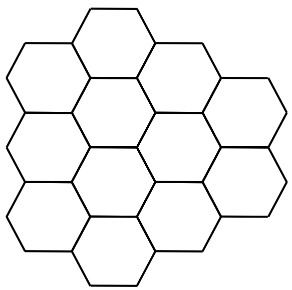
4-cell cluster



3-cell cluster

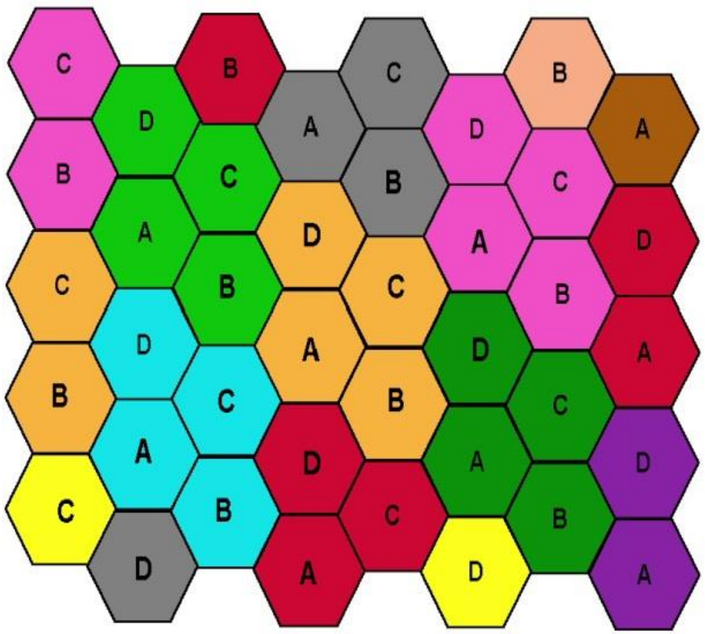


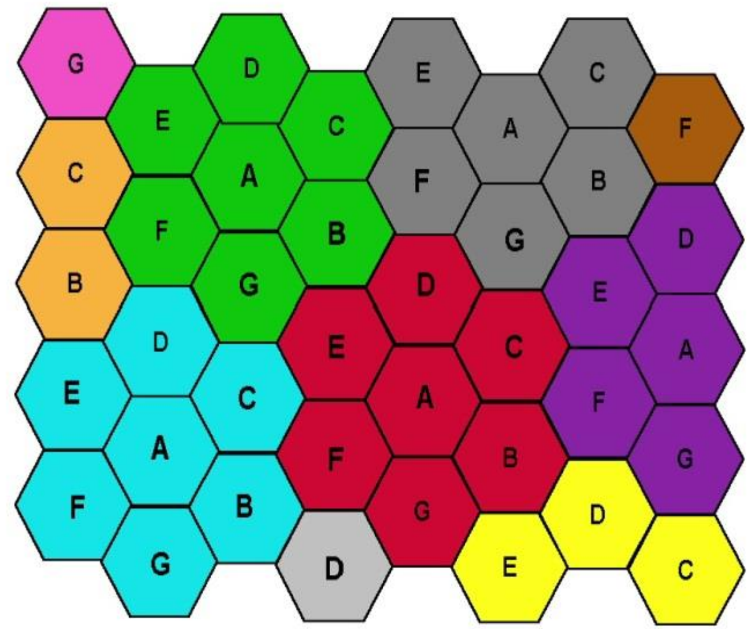
7-cell cluster



12-cell cluster

Cells cluster and frequency reuse pattern





4.2 Interference problem

Carrier to interference power ratio (C/I)

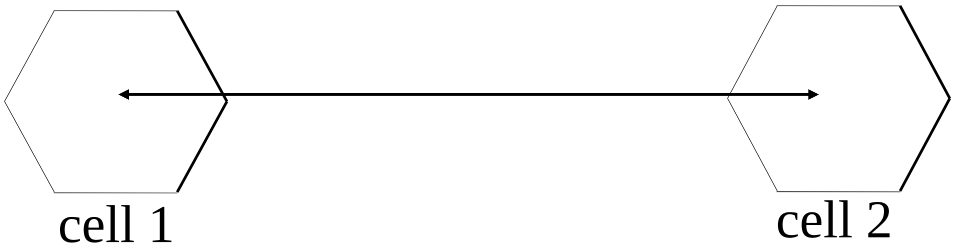
The interference problem results from assigning the same carrier frequency to different cells. The impact of the interference problem is measured by C/I, where C is the carrier power and I is the interfering signal power. Typically the receiver performance requires a minimum signal-to-noise ratio (SNR). Considering the background noise (thermal noise) of N_0, the total noise power can be expressed as $I+N_0$. Therefore to guarantee the required receiver performance, we should have

$$C/(I+N_0) > \text{SNR}_{\text{required.}}$$

Notice that the increase of transmitting power can improve C/N_0, the ratio of signal to background noise, but not C/I.

Example:

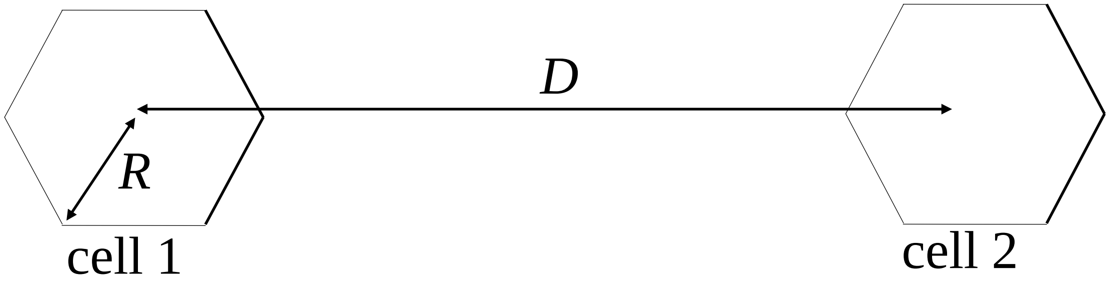
Suppose that we increase the transmitting power of cell 1 by 3dB. Then $C_{cell1}/I_{2\text{-to-}1}$ is improved by 3dB, but $C_{cell2}/I_{1\text{-to-}2}$ is deteriorated by 3dB. To compensate this, we can increase the transmitting power of cell 2 also by 3dB. Overall, this results in no C/I improvement.



If the transmitting power is adjusted to the level that the background noise can be ignored. Such a situation is referred to as "interference limited". Then interference becomes the dominant factor. In this case, further increasing transmission power is not a good option, since it cannot improve the overall C/I for all users.

Example:

In an interference limited situation, we may still adjust the power among different users so as to optimize the distribution of C/I. This is referred to as resource allocation and we will discuss this issue later.



Ratio D/R is called co-channel reuse ratio. It gives an indication of transmission quality. In general, the signal power of a mobile system can be expressed as

$$P = A/r^{\gamma} \tag{4.2}$$

where r is the distance from the transmitter to where the signal is measured, A is a constant and γ is a constant called path loss slope. In free space γ =2. However later we will show that for propagate on earth surface γ=4. In practice γ is between 2 and 4, but sometime it can be as high as 5.

Example:

Consider the special case that a mobile is located at cell 1 border with distance R to its BS. The received signal power at the BS is

$$C = A/R^{\gamma} \qquad (4.3)$$

Assume that cell 2 is using the same frequency. The distance between two cell centers is D. Therefore the interference power I is

$$I = A/D^{\gamma} \qquad (4.4)$$

The signal carrier to interference power ratio at BS of cell 1 is

$$C/I = (D/R)^{\gamma} \rightarrow 10\gamma\log_{10}(D/R) \text{ (dB)} \qquad (4.5)$$

Higher C/I ratio is preferred to guarantee better service performance.

Relationship between *D*/*R* and *N*

Let D be the distance between the centers of two cells using the same frequency in adjacent clusters, and let R be the cell radius. It can be calculated that for an N-cell cluster reuse pattern,

$$D / R = \sqrt{3N} \qquad (4.6a)$$

Example: For a 7-cell cluster, using (4.6a), $D = \sqrt{3 \times 7}\ R$. This is consistent with the result obtained using the figure below (4.1),

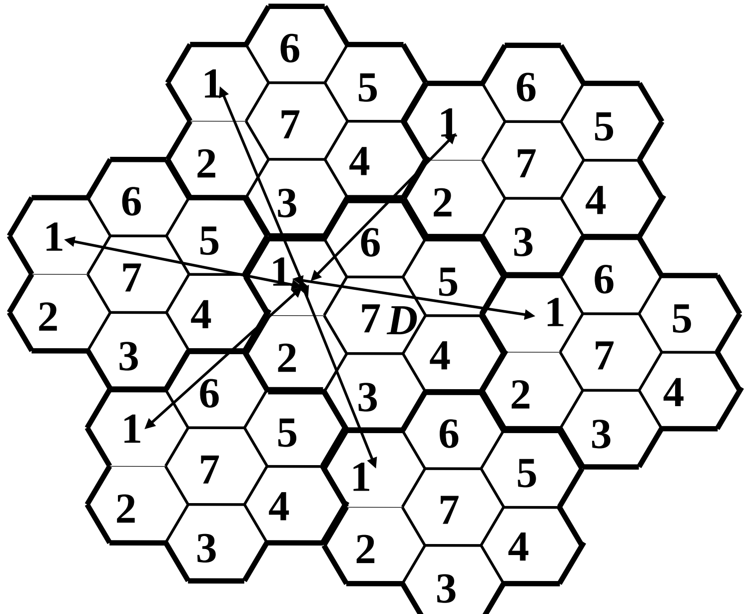
$$D^2 = (2^2 + 1^2 + 1 \times 2) \times 3 \times R^2 = 21R^2$$

Example: What is the minimum distance between the centers of two cells with the same band of frequencies if cell radius is 1 km and the reuse factor is 12?

Solution: $D/R = \sqrt{3N}$

$D = (3 \times 12)^{1/2} \times 1$ km $= 6$ km

Relationship for *C*/*I*, *D*/*R* and *N*



Combine (4.5) and (4.6a) and consider six nearest tiers, we have

$$C/I = \frac{1}{6}(D/R)^{\gamma} = \frac{1}{6}(3N)^{\gamma/2} \qquad (4.6b)$$

The worst-case interference happens when the six interfering users are all located at the nearest borders of their cells. Such distance is approximately *D* – *R*. Therefore from (4.6a), we have the worst-case equivalent of (4.6b).

$$C/I_{\text{worst}} = \frac{1}{6}((D-R)/R)^{\gamma} = \frac{1}{6}(D/R-1)^{\gamma} = \frac{1}{6}(\sqrt{3N}-1)^{\gamma} \qquad (4.7)$$

4.3 Spectrum efficiency

Spectrum efficiency

One definition of spectrum efficiency is user/cell/Hz. This is the number of users per cell if 1Hz bandwidth is assigned to one link of the system.*

Let the bandwidth per user be B Hz. For a 1-cell cluster, the number of users is B^{-1} per cell per Hz, so the spectrum efficiency is

$$\eta = B^{-1} \text{ user/cell/Hz} \qquad (4.8)$$

Let the number of the cells in a cluster is $N_{cluster}$. For an $N_{cluster}$-cell cluster, the spectrum efficiency is reduced to,

$$\eta = \frac{1}{B \times N_{cluster}} \quad \text{user/cell/Hz} \qquad (4.9)$$

Clearly, a large cluster will reduce spectrum efficiency.

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* Notice that normally we assume the up- and down- links occupy the same bandwidth. We may use the words "2×1MHz is assigned to the system" to indicate that 1MHz is assigned to each link.

Impact of D/R and C/I requirement

Recall from (4.6) that (when γ=4)

$$C/I = \frac{1}{6}(D/R)^4 = \frac{1}{6}(3N_{cluster})^2$$

We can rewrite the efficiency in (4.9) as

$$\eta = \frac{3}{B\left(D/R\right)^2} \quad \text{user/cell/Hz}$$
$$= \frac{3}{B\sqrt{6C/I}} \quad \text{user/cell/Hz} \qquad (4.10)$$

Clearly, to increase efficiency, smaller *D*/*R* is preferred (i.e., to reuse the frequency more often). We can accomplish this by reducing the required *C/I*. Therefore it is of vital importance in a cellular system to have efficient transmission techniques (coding, modulation, MIMO …) that can work properly at low *C/I*.

Cellular efficiency in terms of users

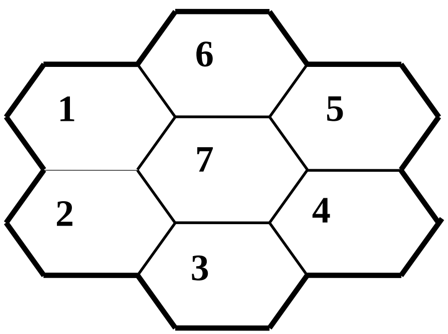
If the bandwidth for each user is fixed, it is convenient to measure spectrum efficiency using user number supported per cell. Then the unit is users/cell. Let the bandwidth per user be *B* Hz. Assume that the available bandwidth is *W* Hz. Then

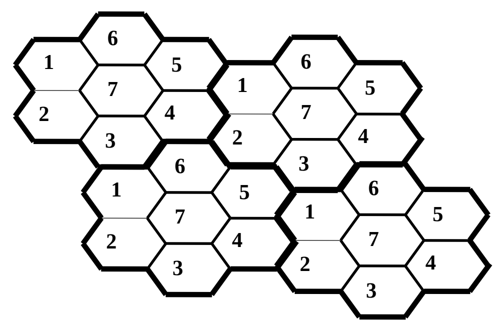
$$N_{\text{users\_per\_cell}} = \frac{W}{B \times N_{cluster}} = \eta W \qquad (4.11)$$

Spectrum efficiency regarding cell size

The spectrum efficiency can also be defined in many other ways, such as user/km^2/Hz (i.e., user number per km^2 for 1Hz per link). This can be obtained by dividing (4.8)-(4.10) by the area of a cell.

Clearly, smaller cells can support more users in a given area. We can see this from the graphs below. Assume both systems below have the same coverage area. The system on the right has four times of the capacity than the one on the left has. (Both D/R and C/I keep unchanged and so the number of users supported by each cell is the same. The number of cells on the right is four times of that on the left.)



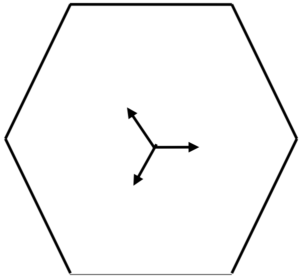


Question: What is the cost for reducing cell size?

Sectorization

Sectorization can further increase spectrum efficiency without increasing base station number. This technique consists of dividing the cell into a number of sectors, each of which is served by a different set of channels and illuminated by a directional antenna. A sector can therefore be considered as a new cell.

It is shown on the right a 120° directional antenna sectorization which is most often used in practical systems.



Example

Consider the following parameters. Minimum SNR at the receiver = 18dB (≈64). Bandwidth (duplexing for a pair of up- and down- links)) = 2×30kHz. Total available duplex channel number = 790.

Let us ignore lognormal and Rayleigh fading. Estimate the efficiency of the system. Let us also ignore the cost of control channel and assume *W/B*=790.

TABLE 1.1 Parameters of Some First-Generation Cellular Standards

Parameters	AMPS	C450	NMT 450	NTT	TACS
Tx Frequency (MHz)					
Mobile	824–849	450–455.74	453–457.5	925–940	890–915
Base Station	869–894	460–465.74	463–467.5	870–885	935–960
Channel bandwidth (kHz)	30	20	25	25	25
Spacing between forward and reverse channels (MHz)	45	10	10	55	45
Speech signal FM deviation	±12	±5	±5	±5	±9.5
Control signal data rate (kbps)	10	5.28	1.2	0.3	8
Handoff decision is based on	Power received at base	Round-trip delay	Power received at base	Power received at base	Power received at base

Solution

The target *C/I*=64. Using (4.6b), we have

$$N_{\text{cluster}} \geq \frac{\sqrt{6 \times 64}}{3} \approx 6.5 = 7$$

From (4.11), we can see that the capacity is

$$N_{\text{users\_per\_cell}} = \frac{1}{7} \times (\text{total number of channels})$$
$$= \frac{790}{7} \approx 113 \ \text{users/cell}$$

The actual capacity is lower er due to many other factors such as fading.

<u>**Note:**</u> Typically, an analogue system requires a relative higher *C/I* ratio than a digital one. With forward error control (FEC) coding, the required *C/I* ratio can be greatly reduced in a digital system. For an analogue system, the cross talk between channels (i.e., your talk may be heard by someone else) can be a serious problem.

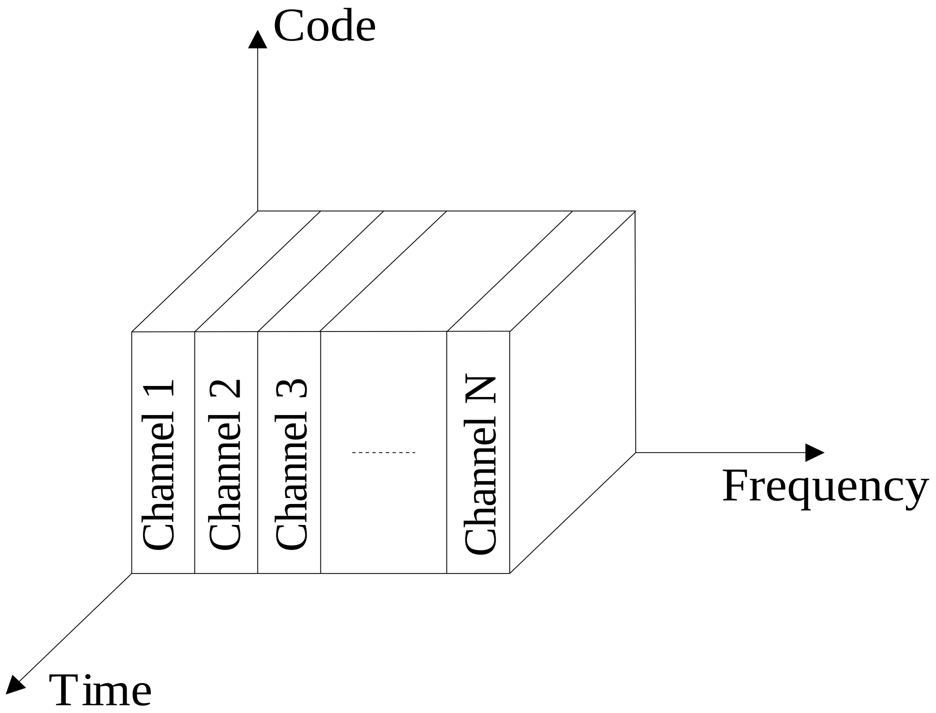
4.4 Multiple access techniques

Multiple access techniques

Multiple access schemes are used to allow many mobile users to share a finite amount of radio spectrum simultaneously. For high quality communications, this must be done without severe degradation in the performance of the system.

FDMA

Frequency division multiple access (FDMA) assigns individual channels to individual users. Each user is allocated a unique frequency band or channel. These channels are assigned on demand to users who request service. During the period of the call, no other user can share the same frequency band.



FDMA

The number of channels that can be simultaneously supported in a FDMA system is given by

$$N_{\text{users\_per\_cell}} = B_t / B_c$$

where B_t is the total spectrum allocation and B_c is the channel bandwidth.

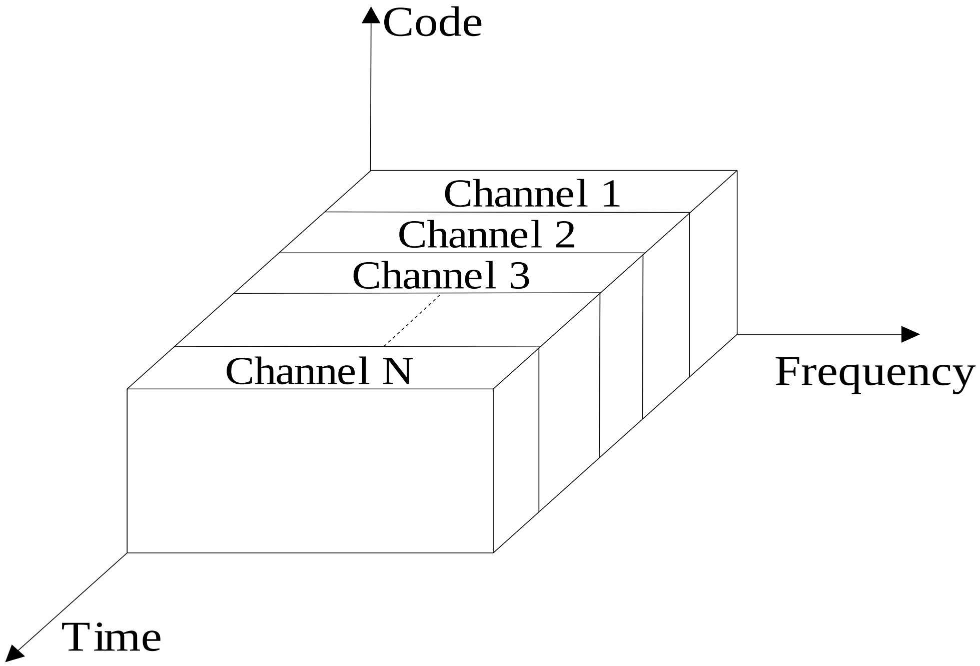
Note:

(1) FDMA alone is used in the first generation analogue systems but is not used in current digital systems.

(2) Typically, an FDMA system requires a quite high $(C/I)_{\text{min}}$ value. (For example, 18dB.) This is due to the narrowband nature of a FDMA system where fading is a very serious problem.

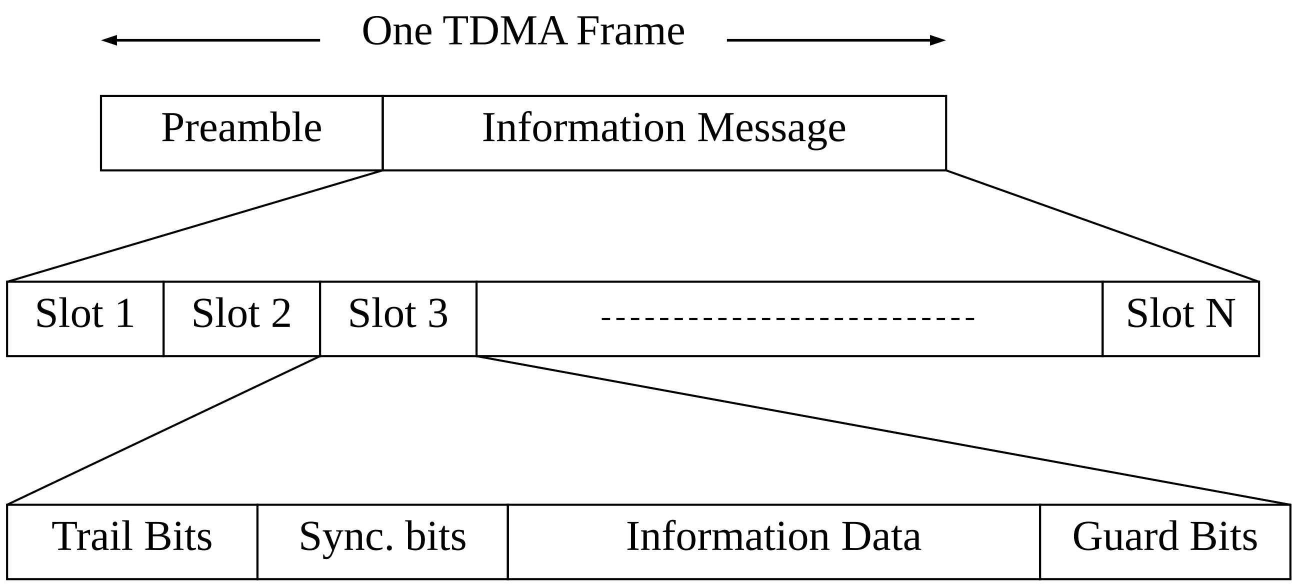
TDMA

Time division multiple access (TDMA) systems divide the radio spectrum into time slots, and in each slot only one user is allowed to either transmit or receive. Each user occupies a cyclically repeating time slot. TDMA systems transmit data in a buffer-and-burst method, thus the transmission for any user is non-continuous.



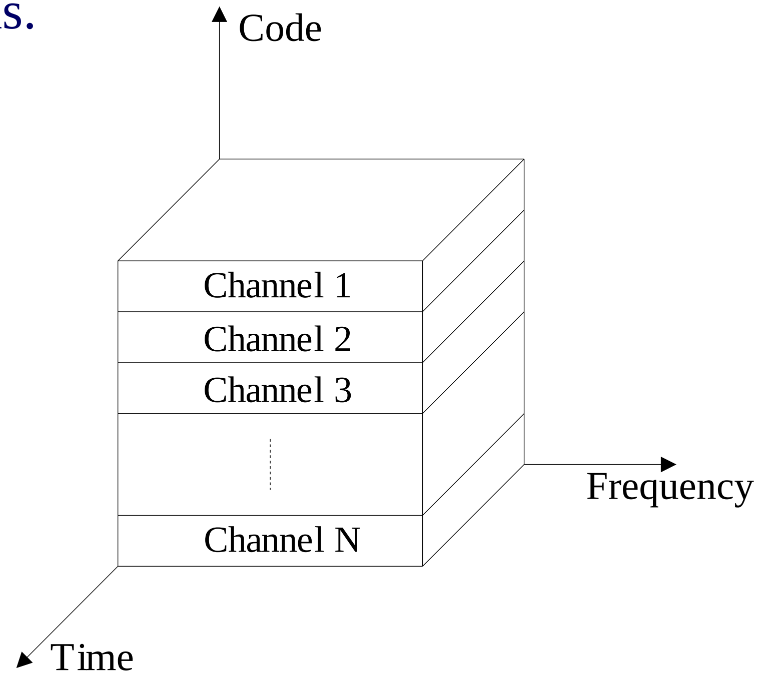
TDMA

The following is an illustration of a TDMA frame structure:



CDMA

In code division multiple access (CDMA) systems, the narrowband message signal is multiplied by a very large bandwidth signal called the spreading signal. The spreading signal is a pseudo-noise code sequence that has a chip rate which is orders of magnitudes greater than the data rate of the message. All users in a CDMA system use the same frequency and may transmit simultaneously. Each user has its own pseudorandom codeword that is approximately orthogonal to all the other codewords.



CDMA

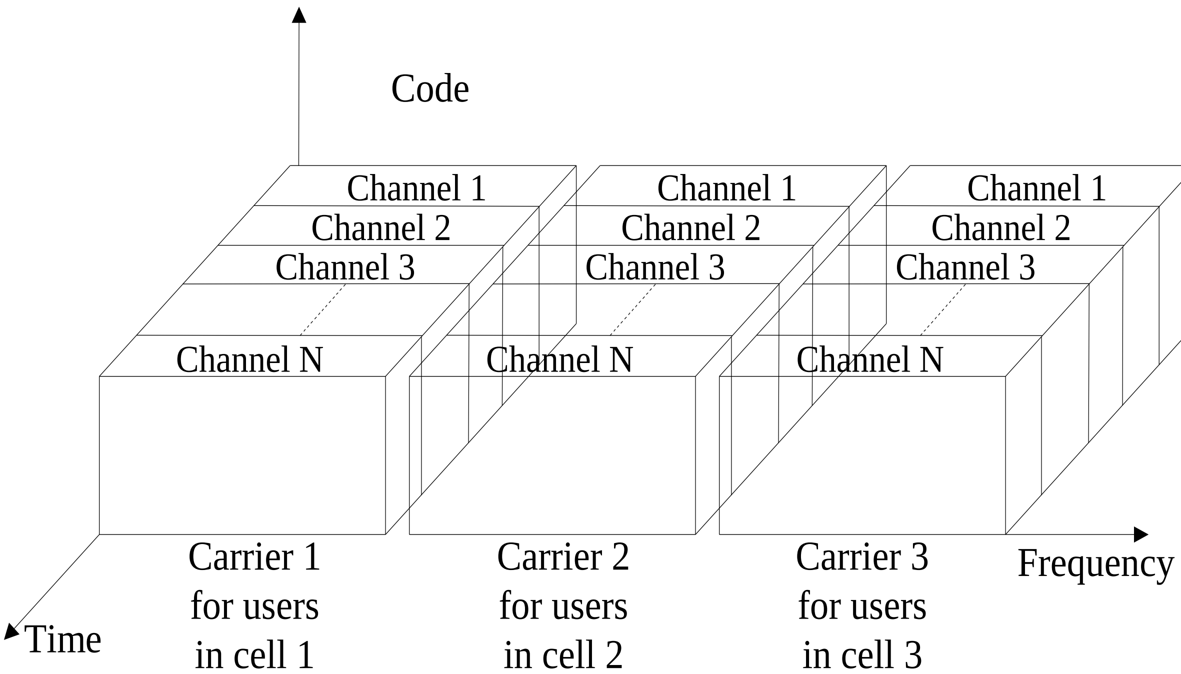
The receiver performs a time correlation operation to detect only the specific desired codeword. All other codewords appear as noise due to de-correlation.

CDMA is 3G propositions. The capacity of a CDMA system is calculated quite differently from the discussion earlier, and we will discuss this in detail later .

A CDMA system requires a lower $(C/I)_{\mathrm{min}}$ value than FDMA and TDMA systems. (For example, 7dB for CDMA, 9dB for TDMA and 18dB for FDMA.) This is due to the wideband nature of a CDMA system where multi-path can provide diversity against fading.

Hybrid FDMA-TDMA

FDMA and are jointly used in 2G. This is because hybrid filtering and time slotting are convenient for cellular design. For example, we can divide the overall spectrum into several sub-bands called carriers. TDMA is used within a cell. FDMA is used among cells.



Hybrid FDMA-TDMA

In a hybrid system, FDMA using filtering is used as a low-cost channel separation technique. TDMA allows flexible resource allocation.

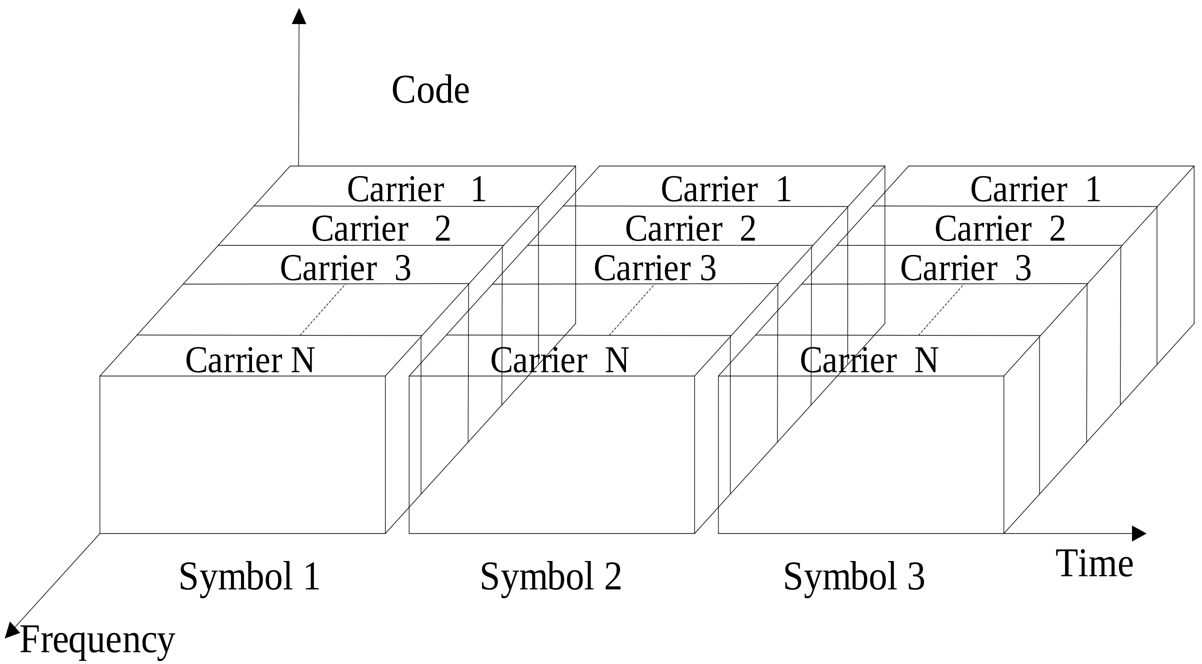
Example: Consider a mixed FDMA-TDMA/FDD system with a spectrum of 25MHz for the forward link (the bandwidth of each channel is 200kHz). Eight time slots are supported on each channel. The number of simultaneous users that can be accommodated is given as

$$N_{\text{users\_per\_cell}} = \frac{8 \times 25\text{MHz}}{N_F \times 200\text{KHz}} = \frac{1000}{N_F}$$

where N_F is the frequency reuse factor. This is approximately the situation with 3G.

OFDM

ODMA is used in 4G and 5G. It can be seen as a discrete form of hybrid TDMA and TDMA system. The total time-frequency resource is divided into discrete grid points of time-domain symbols on frequency domain carriers. These resource units are then allocated among different users.



Chapter 4 summary

Interference problem.

$$C/I = (D/R)^{\gamma} \rightarrow 10\gamma\log_{10}(D/R)\ \text{(dB)}$$

Regular clusters.

$$D^2 = (\mathrm{n}_{\mathrm{v}}^2 + n_u^2 + n_v n_u)3R^2$$

Relationship among *D*, *R* and *N*

$$D/R = \sqrt{3N}$$

Co-channel reuse ratio *D/R* with γ=4,

$$C/I = \frac{1}{6}(D/R)^4 = \frac{1}{6}(3N)^2$$

Chapter 4 summary

Spectrum efficiency

$$\eta = \frac{3}{B\left(D/R\right)^2} \quad \text{user/cell/Hz}$$

$$= \frac{3}{B\sqrt{6C/I}} \quad \text{user/cell/Hz}$$

Cellular efficiency in terms of users

$$N_{\text{users\_per\_cell}} = \frac{W}{B \times N_{cluster}} = \eta W \quad \text{users/cell}$$

An interference limited situation is when thermal noise can be ignored and cross user interference becomes the dominate factor.