Chapter 5

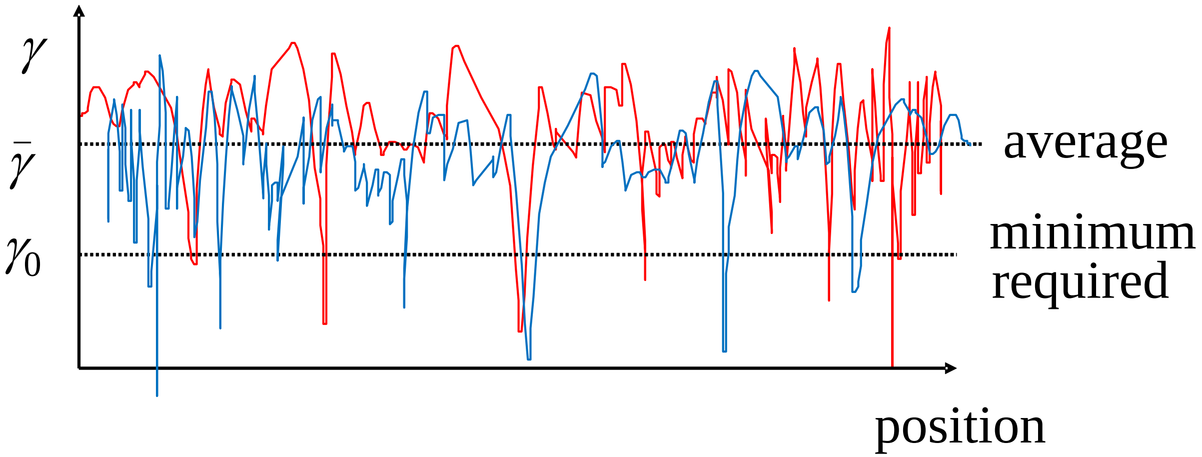
Multiple Antenna Techniques

Introduction

Introduction

In mobile systems, a key requirement is not to increase the total transmitted power, since more transmitted power generally means more interference. The multiple antenna techniques can increase received power without increasing transmitted power. This achieves through improved power gain.

An important concept covered in this chapter is antenna diversity to overcome fading effect. This is because the probability that all antennas fade simultaneously is relatively small if they are separated by sufficient distances, as illustrated by the figure below. No CSIT is required in this case.



Introduction

We will focus on relatively simple single-input multiple-output (SIMO) and multiple-input single-output (MISO) systems. We will only briefly outline the multiple-input multiple-output (MIMO) principle that has been identified as a key technology for future cellular systems.

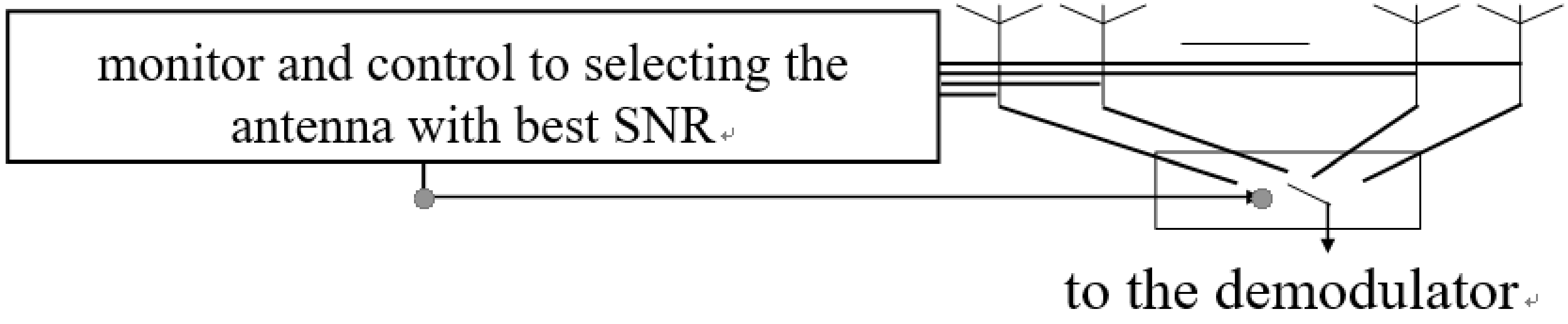
Chapter outline:

Part 1 Antenna combining techniques
Part 2 Space-time coding
Part 3 MIMO

5.1 Antenna combining techniques

Selection combining

Combing techniques refer to the methods of utilizing the signals from different antennas. A common technique is switched combining, in which the best signal is selected from different antennas. In theory the best signal is the one with maximum SNR. In practice, however, it is difficult to judge and normally the strongest signal is selected.



Selection combining

Define

$$\gamma_i = \frac{\text{local mean signal power at antenna } i}{\text{mean noise power}} = \frac{p_i}{\sigma^2}$$

where i=1, 2, …, M for M antennas. We assume that p_i is exponentially distributed (Rayleigh fading) and σ^2 is a constant. Then γ_i has an exponential pdf

$$p_{\text{single antenna}}(\gamma_i) = \bar{\gamma}^{-1} e^{-\frac{\gamma_i}{\bar{\gamma}}}, \quad \text{for all} \quad \gamma_\text{i} > 0$$

The outage is defined as

$$\Pr(\gamma_i < \gamma_0)$$

This is the probability that the signal from one antenna is below a threshold value γ_0. The outage can be calculated using a cumulated distribution function (cdf) given below.

$$P_{\text{single antenna}}(\gamma_0) = \Pr(\gamma_i < \gamma_0) = \int_0^{\gamma_0} p(\gamma_i) d\gamma_i = 1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}$$

Selection combining

The outage probability that the best signal among M antennas falling below a threshold value γ_0 is given by the following cdf:

$$P_{M\ \text{antennas}}(\gamma_0) = \Pr(\gamma_{\max} < \gamma_0) = (\ \Pr(\gamma_i < \gamma_0)\ )^M = \left(1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}\right)^M \quad (5.3)$$

It characterizes switching combining with M antennas, average SNR $\bar{\gamma}$ and threshold γ_0

We can also get a pdf by taking differentiation to cdf in (5.3).

$$p_{M\ \text{antennas}}(\gamma_0) = dP_{M\ \text{antennas}}(\gamma_0)/d\gamma_0 = \left(Me^{-\frac{\gamma_0}{\bar{\gamma}}}/\bar{\gamma}\right)\left(1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}\right)^{M-1}$$

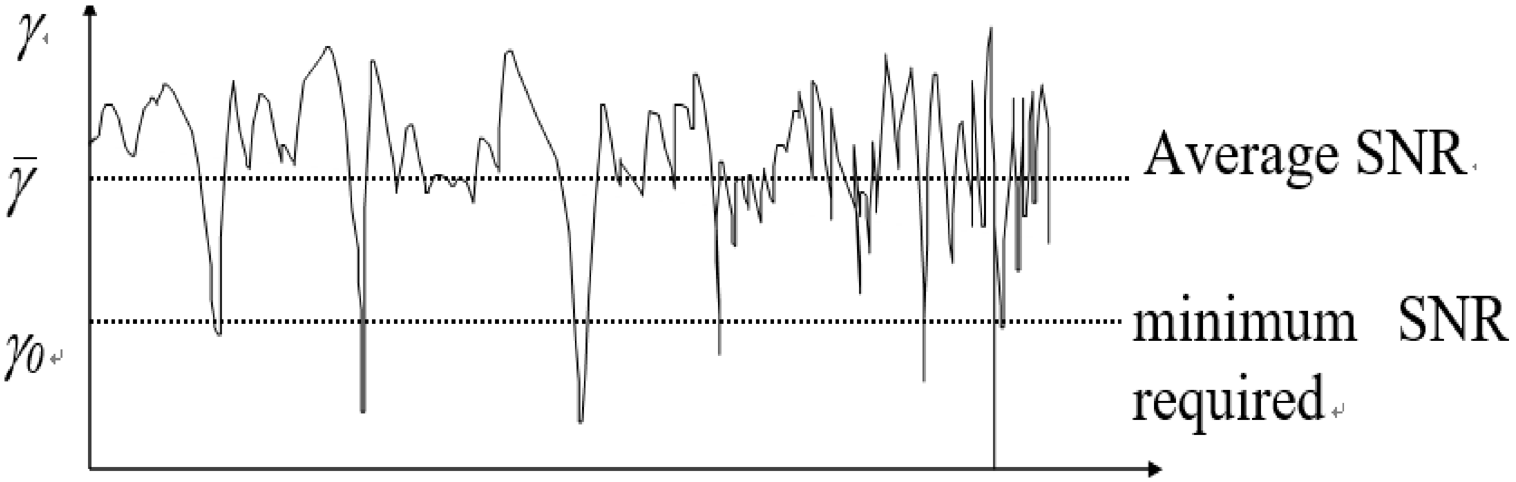
The average SNR at the combining output is given by

$$\int_0^\infty \gamma_0 p_{M\ \text{antennas}}(\gamma_0) d\gamma_0 = \int_0^\infty \gamma_0 \left(Me^{-\frac{\gamma_0}{\bar{\gamma}}}/\bar{\gamma}\right)\left(1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}\right)^{M-1} d\gamma_0 \quad (5.4)$$

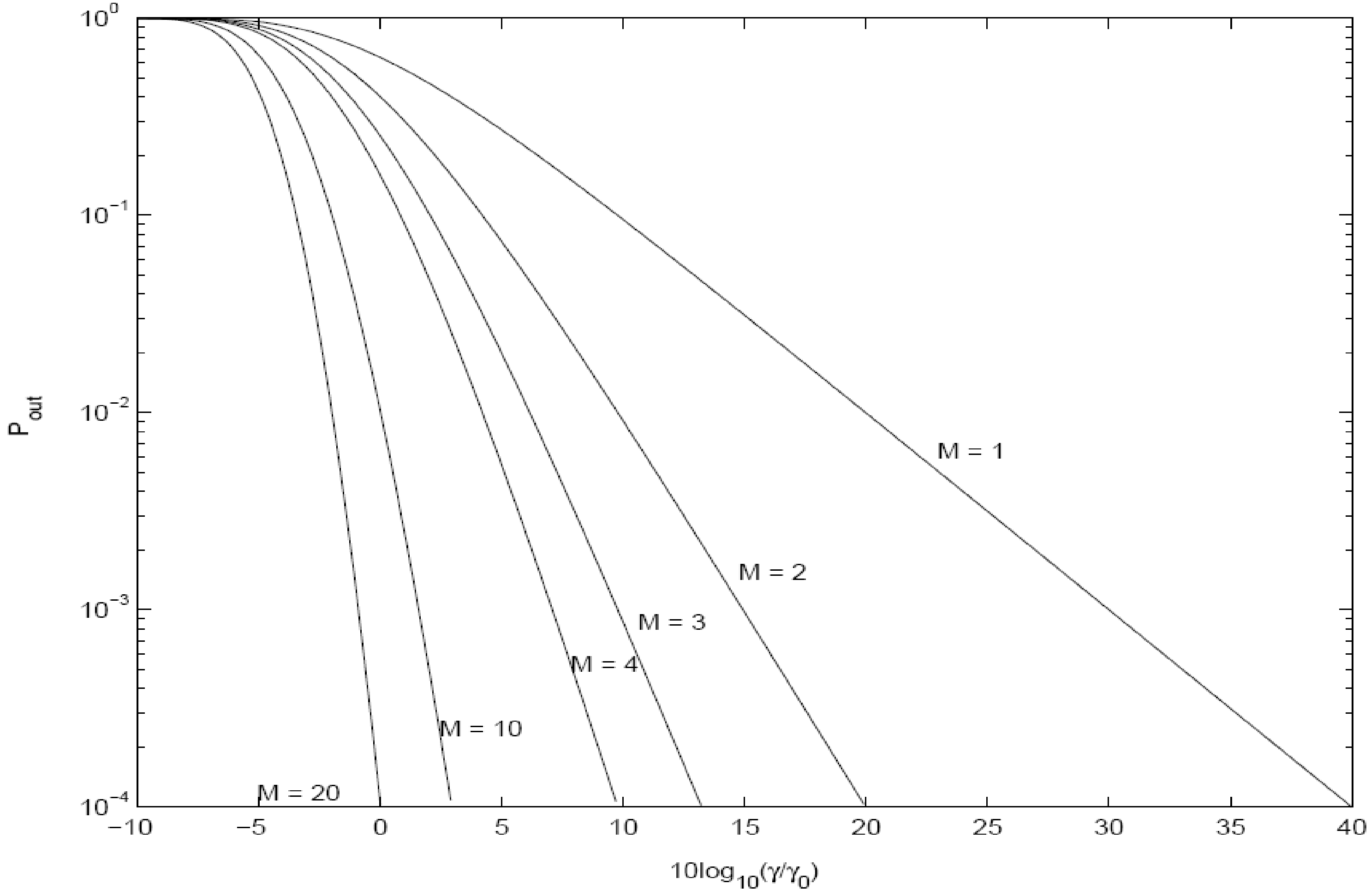
$$= \bar{\gamma}\sum_{i=1}^{M}\frac{1}{i}$$

Selection combining

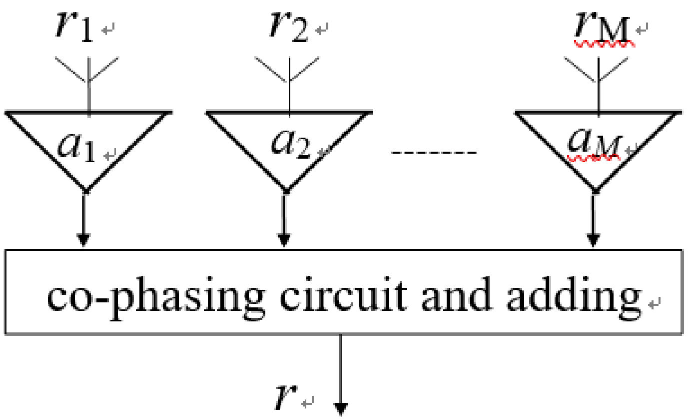
$$P_{M\ \text{antennas}}(\gamma_0) = \Pr(\gamma_{\max} < \gamma_0) = (\ \Pr(\gamma_i < \gamma_0)\ )^M = \left(1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}\right)^M$$



Outage probability of selection combining in Rayleigh fading



Gain combining



With gain combining, the signal from the *i*th antenna is scaled by a complex coefficient a_i (meaning adjustment on both amplitude and phase) and summed. The received signal for the *i*th antenna is given in its phasor form as

$$r_i = h_i d + \eta_i \qquad (5.6)$$

where h_i is a channel coefficient, d is the transmitted signal and η_i is a noise sample with variance σ^2. For simplicity, assume the same noise power for every antenna *i*. The output of the combiner is

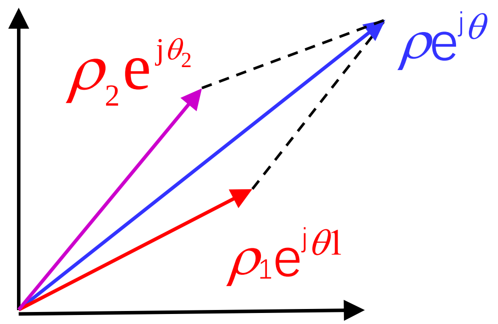
$$r = \sum_{i=1}^{M} a_i r_i = \left(\sum_{i=1}^{M} a_i h_i \right) d + \sum_{i=1}^{M} a_i \eta_i \qquad (5.7)$$

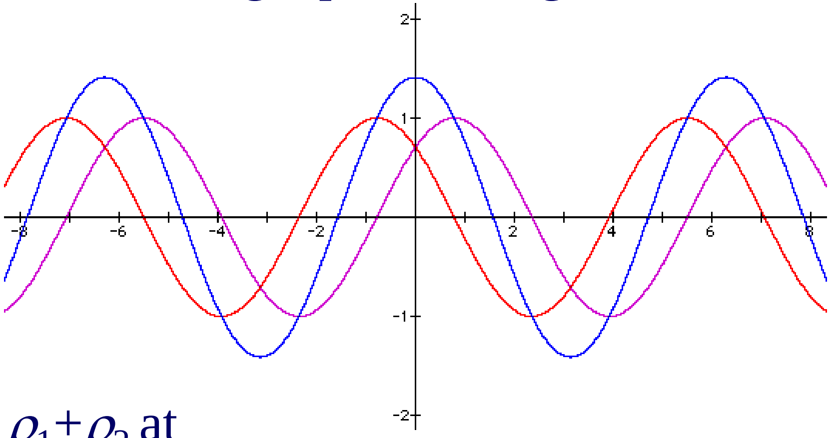
Background: coherent signal adding

Consider

$$\rho\cos(2\pi ft+\theta)=\rho_1\cos(2\pi ft+\theta_1)+\rho_2\cos(2\pi ft+\theta_2)$$

Given ρ_1, ρ_2, θ_1 and θ_2, we can find ρ and θ using a phasor diagram:





Clearly, $|\rho|\le|\rho_1+\rho_2|$ and max(ρ) = ρ_1+ρ_2 at

$$\theta_1=\theta_2.$$

We say that the addition is coherent when θ_1=θ_2. Coherent addition is crucial for power maximization.

Adding non-coherent and coherent variables

We now discuss more general cases of adding multiple variables

$$\Sigma = \sum_{i=1}^{M} z_i , \qquad (5.8a)$$

where $\{z_i\}$ are phasors:

$$z_i = \rho_i e^{j2\pi ft+\theta_i} \qquad (5.8b)$$

We consider two extreme cases:

(i) $\{z_i\}$ are independent random variables with zero means. In this case, from probability theory,

$$\mathrm{E}\left(\left|\Sigma\right|^2\right) = \mathrm{Var}\left(\Sigma\right) = \sum_{i=1}^{M} \mathrm{Var}\left(z_i\right) = \sum_{i=1}^{M} \mathrm{E}\left(\left|z_i\right|^2\right) \qquad (5.8c)$$

This is referred to as non-coherent adding. In particular, if all $\{z_i\}$ have the same average power, then

$$\mathrm{E}\left(\left|\Sigma\right|^2\right) = M \cdot \mathrm{E}\left(\left|z_i\right|^2\right) \qquad (5.8d)$$

Adding non-coherent and coherent variables

The phases of $\{z_i\}$ are aligned, i.e., $\theta_1 = \theta_2 = \ldots = \theta_M$. In this case,

$$\left|\Sigma\right|^2 = \left(\sum_{i=1}^{M} \left|z_i\right|\right)^2 \qquad (5.8e)$$

This is referred to as coherent adding. In particular, if all $\{z_i\}$ have the same power, then

$$\left|\Sigma\right|^2 = M^2 \cdot \left|z_i\right|^2 \qquad (5.8f)$$

From (5.8d) and (5.8f), coherent adding has a power gain of M times than non-coherent adding.

The advantage of coherent adding can also be seen from the following general inequality:

$$\left|\sum_{i=1}^{M} z_i\right|^2 \le \left(\sum_{i=1}^{M} \left|z_i\right|\right)^2 \qquad (5.8g)$$

The above shows the optimality of coherent adding.

Phase alignment

We now consider a set of variables $\{z_i = a_i h_i\}$. Clearly, to achieve coherent adding, we can properly choose $\{a_i\}$ to align the phases of $\{a_i h_i\}$.

Example: Let $\{a_i = \pm 2\}$ and $\{h_i = \pm 2\}$. We have two cases:

Non-coherent adding: All $\{a_i\}$ and $\{h_i\}$ are independent. Then

$$\mathrm{E}\left(\left|\sum_{i=1}^{M} a_i h_i\right|^2\right) = \sum_{i=1}^{M} \mathrm{E}\left(\left|a_i h_i\right|^2\right) = M \times (2 \times 2)^2 = 16M$$

Coherent adding: We choose $a_i = h_i$ for every i. Then

$$\mathrm{E}\left(\left|\sum_{i=1}^{M} a_i h_i\right|^2\right) = \mathrm{E}\left(\left(\sum_{i=1}^{M} \left|a_i h_i\right|\right)^2\right) = \left(M \times (2 \times 2)\right)^2 = 16M^2$$

Note: In the above, non-coherent adding leads to "power adding" while coherent adding leads to "magnitude adding". The former may involve signal cancelation since the signs of $\{a_i h_i\}$ can be different, while the latter always involve signal enhancement since the signs of $\{|a_i h_i|\}$ are all non-negative.

Equal gain combining (EGC)

Let

$$h_i = |h_i| e^{j\theta_i} \qquad \text{for } i=1, 2, \ldots, M \quad .$$

With EGC, the combining coefficients are

$$a_i = e^{-j\theta_i} \qquad \text{for } i=1, 2, \ldots, M \quad .$$

Clearly,

$$|a_i|=1 \qquad \text{for } i=1, 2, \ldots, M.$$

The combining output is

$$r = \sum_{i=1}^{M} a_i r_i = \left(\sum_{i=1}^{M} |h_i| \right) d + \sum_{i=1}^{M} e^{-j\theta_i} \eta_i$$

The above operation is referred to as "phase alignment" or "co-phasing". It results in coherent adding. In practice, each $e^{-j\theta_i}$ is realized by a time delay circuit of θ_i. We will come back to this later.

Power gain with EGC

EGC improves SNR statistically. To see this, define

$$\text{effective power gain} = \frac{\text{SNR at receiver output (after combinging)}}{\text{SNR at the transmitter}}$$

Let $P = |d|^2$. Then

SNR at the transmitter is P/σ^2.

$$\text{signal power after combiner} = \left(\sum_{i=1}^{M} |h_i|\right)^2 P$$

$$\text{noise power after combiner} = \mathrm{E}\left(\left|\sum_{i=1}^{M} e^{-j\theta_i}\eta_i\right|^2\right) = M\sigma^2$$

Note that σ^2 is actually the noise power at the receiver. Thus P/σ^2 is only a reference value that is useful in measuring the effects of the channel and receiver.

SNR after the combiner: $\left(\sum_{i=1}^{M} |h_i|\right)^2 P / \left(M\sigma^2\right)$

SNR gain: $\left(\sum_{i=1}^{M} |h_i|\right)^2 / M$

Example:

EGC is not optimal. Let $r_1=1+\eta_1$ and $r_2=-2+\eta_2$. Compare 3 methods:

Method 1: $a_1=1$, $a_2=1$,

$r = r_1+r_2+(\eta_1+\eta_2)= -1+(\eta_1+\eta_2)$,
SNR = $(-1)^2/2\sigma^2 =1/2\sigma^2$.

Method 2(EGC): $a_1=1$, $a_2=-1$,

$r = r_1-r_2+(\eta_1-\eta_2)= 3+(\eta_1-\eta_2)$,
SNR = $9/2\sigma^2 =4.5/\sigma^2$.

Method 3: (MRC) $a_1=1$, $a_2=-2$,

$r = r_1-2r_2+(\eta_1-2\eta_2) = 5+(\eta_1-2\eta_2)$,
SNR = $25/5\sigma^2=5/\sigma^2$.

Time domain derivation for the co-phase operation

We now summarize two different approaches to the derivation of the co-phase operation.

Time domain approach: Consider a transmitted cosine signal (ignoring noise)

$$s(t) = d\cos(2\pi f_c t)$$

Let the received signal on antenna i be

$$r_i(t) = |h_i| d\cos\left(2\pi f_c t + \theta_i\right)$$

where $|h_i|$ is the amplitude gain and θ_i is the phase change due to delay. The co-phasing operation adds a phase of $-\theta_i$ to $r_i(t)$ and

$$r(t) = \sum_{i=1}^{M} |h_i| d\cos\left(\left(2\pi f_c t + \theta_i\right) - \theta_i\right) \tag{5.10}$$

This is to align the phases of different $r_i(t)$.

Phasor domain derivations for the co-phase operation

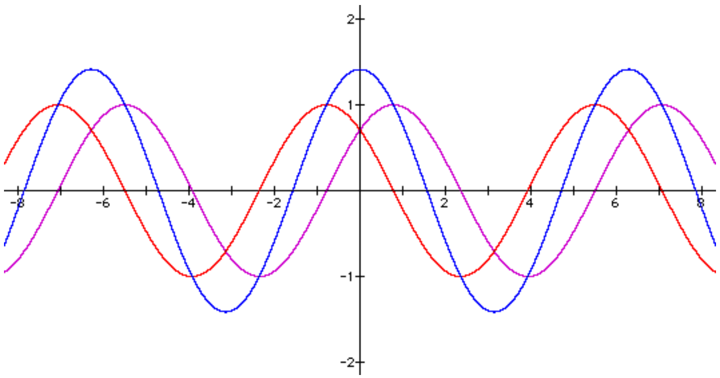
<u>Phasor domain approach</u>: Let the phasor of $s(t)$ be $s=d$. The phasor for $r_i(t)$ is

$$r_i = h_i d = |h_i| e^{j\theta_i} d$$

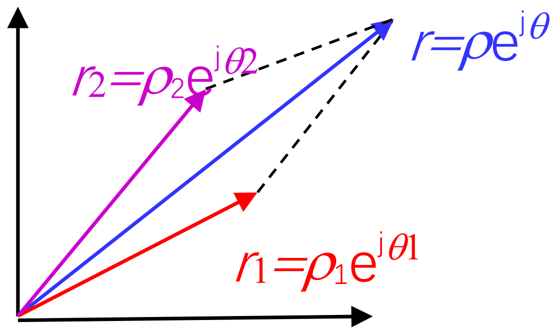
The phasor representation of (7.10) is

$$r = \sum_{i=1}^{M} e^{-j\theta_i} h_i d = \sum_{i=1}^{M} |h_i| d$$

Each corresponds to a proper delay in the time domain.



Time domain

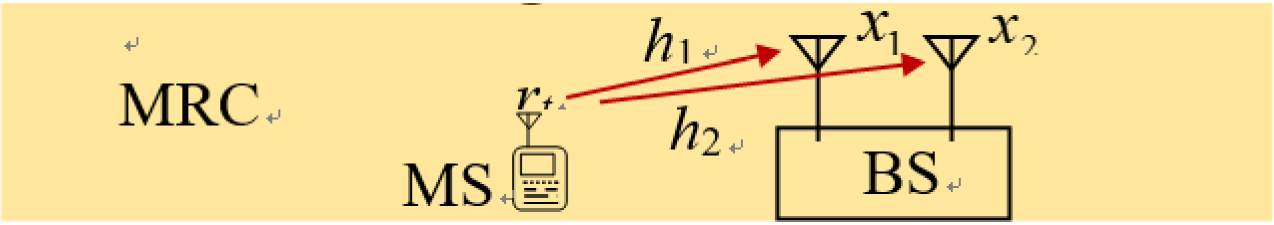


Phasor domain

In the above figures, we can see that ρ is maximized if θ_1 and θ_2 are equal. This is the purpose of the co-phasor operation, which cancels out their difference.

Clearly, the use of phasors provides a more concise approach.

Maximum ratio combining (MRC)



With MRC, we select $a_i = \bar{h}_i$. Substituting this into (5.7), we have

$$r = \sum_{i=1}^{M} \left(|h_i|^2 d + \bar{h}_i \eta_i \right)$$

The SNR in r is (with $P = |d|^2$):

$$SNR_r = \frac{\left(\sum_{i=1}^{M} |h_i|^2 \right)^2 P}{\left(\sum_{i=1}^{M} |\bar{h}_i|^2 \right) \sigma^2} = \sum_{i=1}^{M} \frac{|\bar{h}_i|^2 P}{\sigma^2} = \sum_{i=1}^{M} SNR_i \qquad (5.11)$$

where SNR_i is the SNR for antenna i,

$$SNR_i = |h_i|^2 P / \sigma^2$$

Thus, with MRC, the output SNR is the sum of individual SNRs.

The optimality of MRC

Recall (5.7): $r = \sum a_i \left(h_i d + \eta_i \right)$. For arbitrary $\{a_i\}$, the SNR after combining is given by

$$SNR_r = \frac{\left| \sum_{i=1}^{M} a_i h_i d \right|^2}{E\left(\left| \sum_{i=1}^{M} a_i \eta_i \right|^2 \right)} = \frac{\left| \sum_{i=1}^{M} a_i h_i \right|^2 P}{\sum_{i=1}^{M} \left| a_i \right|^2 \sigma^2}$$

$$\le \frac{\left(\sum_{i=1}^{M} \left| a_i \right|^2 \right) \left(\sum_{i=1}^{M} \left| h_i \right|^2 \right) P}{\sum_{i=1}^{M} \left| a_i \right|^2 \sigma^2} = \sum_{i=1}^{M} \left| h_i \right|^2 P / \sigma^2 = \sum_{i=1}^{M} SNR_i \qquad (5.12)$$

where we have used the Cauchy–Schwarz inequality. From (5.12), the output SNR for any $\{a_i\}$ cannot exceed ΣSNR_i. Thus MRC is optimal with respect to SNR maximization.

Power gain with MRC

SNR after the combiner:	$\sum_{i=1}^{M} SNR_i = \sum_{i=1}^{M} \|h_i\|^2 P / \sigma^2$
SNR at the transmitter:	P/σ^2
SNR gain:	$\sum_{i=1}^{M} \|h_i\|^2$

Cauchy–Schwarz inequality

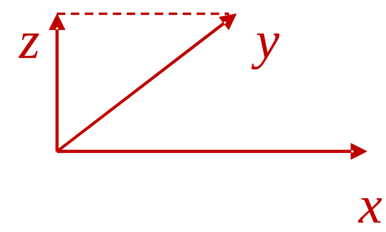
The Cauchy–Schwarz inequality states that

$$\left(\sum_{i=1}^{n}|x_i|^2\right)\left(\sum_{i=1}^{n}|y_i|^2\right) \geq \left|\sum_{i=1}^{n}|x_i\bar{y}_i|\right|^2$$

or
$$|\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}||\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}| \geq |\boldsymbol{x}\boldsymbol{y}^{\mathrm{H}}|^2$$

Proof: If x=0 it is clear that we have equality. For $x\neq 0$, let

$$\boldsymbol{z} = \boldsymbol{y} - \left(\frac{\boldsymbol{y}\boldsymbol{x}^{\mathrm{H}}}{\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}}\right)\boldsymbol{x} \qquad \text{(i)}$$



so
$$\boldsymbol{z}\boldsymbol{x}^{\mathrm{H}} = \boldsymbol{y}\boldsymbol{x}^{\mathrm{H}} - \left(\frac{\boldsymbol{y}\boldsymbol{x}^{\mathrm{H}}}{\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}}\right)\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}} = \mathbf{0}$$

We can equivalently write (i) as

$$\boldsymbol{y} = \frac{\boldsymbol{y}\boldsymbol{x}^{\mathrm{H}}}{\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}}\boldsymbol{x} + \boldsymbol{z}$$

$$\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}} = \left(\frac{\boldsymbol{y}\boldsymbol{x}^{\mathrm{H}}}{\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}}\right)^2 \boldsymbol{x}\boldsymbol{x}^{\mathrm{H}} + \boldsymbol{z}\boldsymbol{z}^{\mathrm{H}} \geq \frac{\left(\boldsymbol{y}\boldsymbol{x}^{\mathrm{H}}\right)^2}{\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}}$$

Note the orthogonality between z and x.

or
$$\left(\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}\right)\left(\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}\right) \geq \left(\boldsymbol{y}\boldsymbol{x}^{\mathrm{H}}\right)^2$$

Chi-square distribution

In MRC, the output SNR is the sum of the SNR on M antennas. The latter are random variables. The SNR on each antenna is given by

$$SNR_i = |h_i|^2 P / \sigma^2$$

With Rayleigh fading, each h_i is Gaussian distributed and $|h_i|^2$ is exponentially distributed. The SNR after combining is

$$\sum_{i=1}^{M} SNR_i = \sum_{i=1}^{M} |h_i|^2 P / \sigma^2$$

Let us ignore the common factor P/σ^2. Define

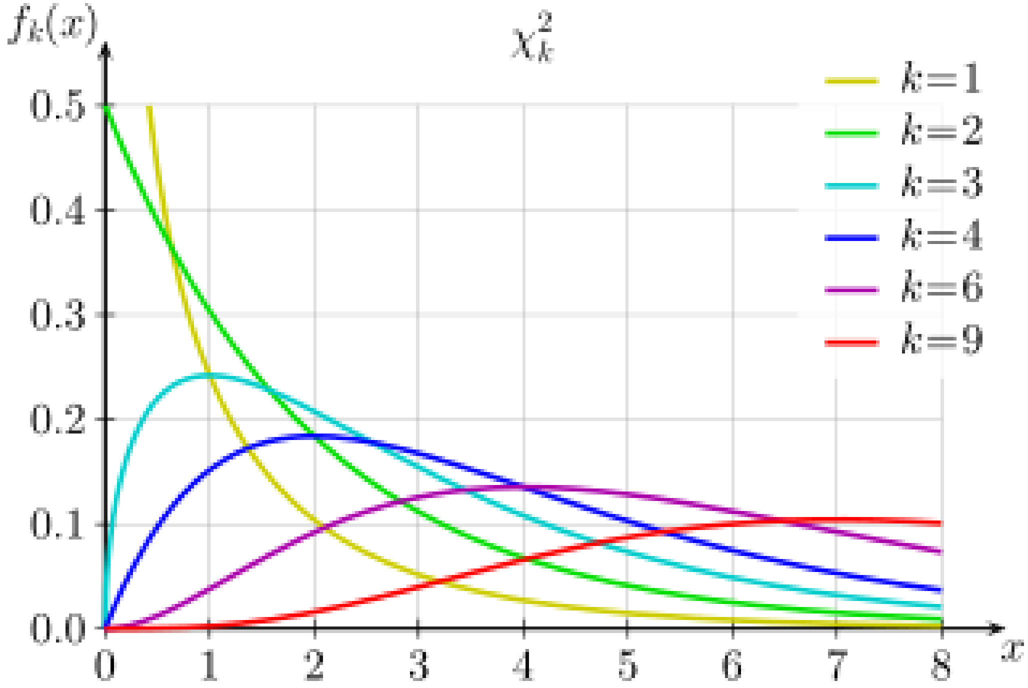
$$y = \sum_{i=1}^{M} |\xi_i|^2 = \sum_{i=1}^{M} \left(|\mathrm{Re}(\xi_i)|^2 + |\mathrm{Im}(\xi_i)|^2 \right)$$

where each $\mathrm{Re}(\xi_i)$ or $\mathrm{Im}(\xi_i)$ is Gaussian distributed with zero mean. It can be shown that y is a chi-square distributed (or χ^2 distributed). Its PDF is given by (with $\mathrm{Var}(\mathrm{Re}(\xi_i))= \mathrm{Var}(\mathrm{Im}(\xi_i))=1$)

$$f(y) = \begin{cases} \dfrac{y^{M-1} e^{y/2}}{2^M \Gamma(M)} & y > 0 \\ 0 & y \le 0 \end{cases}$$

and $2M$ is called the free-degree of the variable.

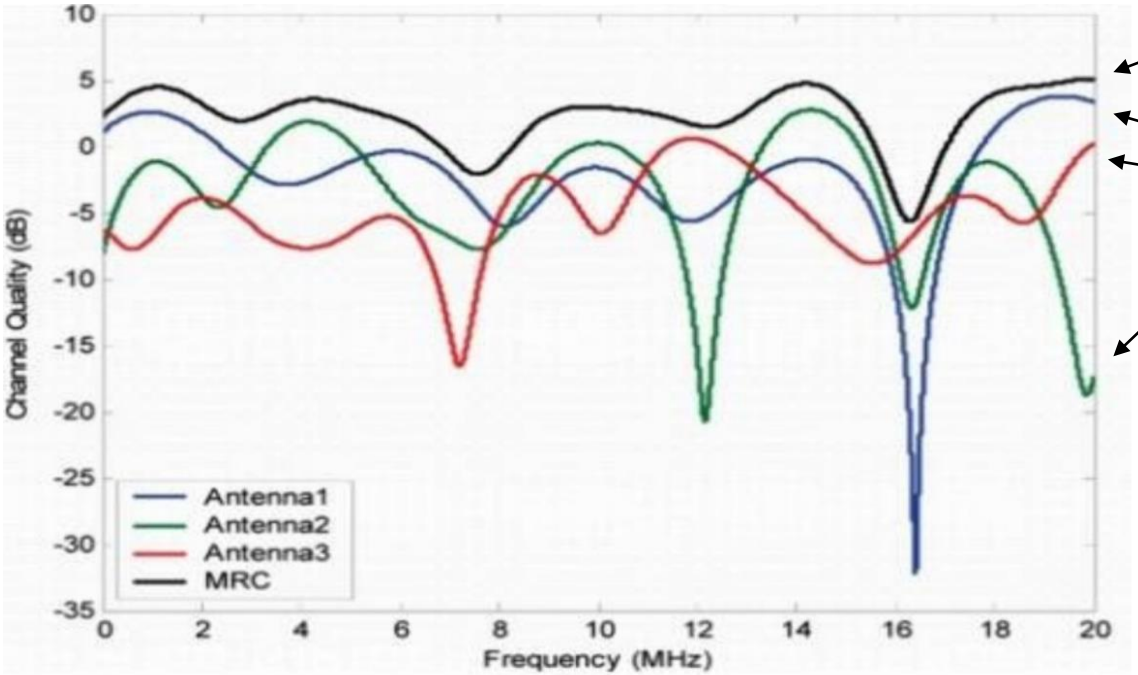
Chi-square distribution



The exponential distribution, i.e., Rayleigh fading, is a Chi-square distribution with free-degree of 2.

Chi-square distribution

The following is an example for three individual antennas and MRC, which combining with chi-square distribution. We can see that after combining, the probability of deep fades is significantly reduced. Thus gain combination technique can increase average power as well as alleviate fading effect.



χ^2 with free-degree of 6

Rayleigh (χ^2 with free-degree of 2)

Summary: non-coherent combining, EGC and MRC

For non-coherent combining refers to the operation below: (In general, “non-coherent” means ignoring phase, while “coherent” implying operation on phase.)

$$r = \sum_{i=1}^{M} r_i = \sum_{i=1}^{M} \left(h_i d + \eta_i \right)$$

From (5.8c), it can be shown that

$$SNR_{\text{non-coherent combining}} = \sum_{i=1}^{M} \left| h_i \right|^2 P / \left(M \sigma^2 \right)$$

Note that here noise is increased by M times after adding. From our earlier discussions, we have power gains:

EGC: $\left(\sum_{i=1}^{M} \left| h_i \right| \right)^2 / M$

MRC: $\sum_{i=1}^{M} \left| h_i \right|^2$

Non-coherent: $\sum_{i=1}^{M} \left| h_i \right|^2 / M$

Thus MRC is better than EGC. The proof can be obtained using the Cauchy–Schwarz inequality. It is also seen that MRC outperforms non-coherent combining by M times when all $\{h_i\}$ have equal power.

Performance of gain combining

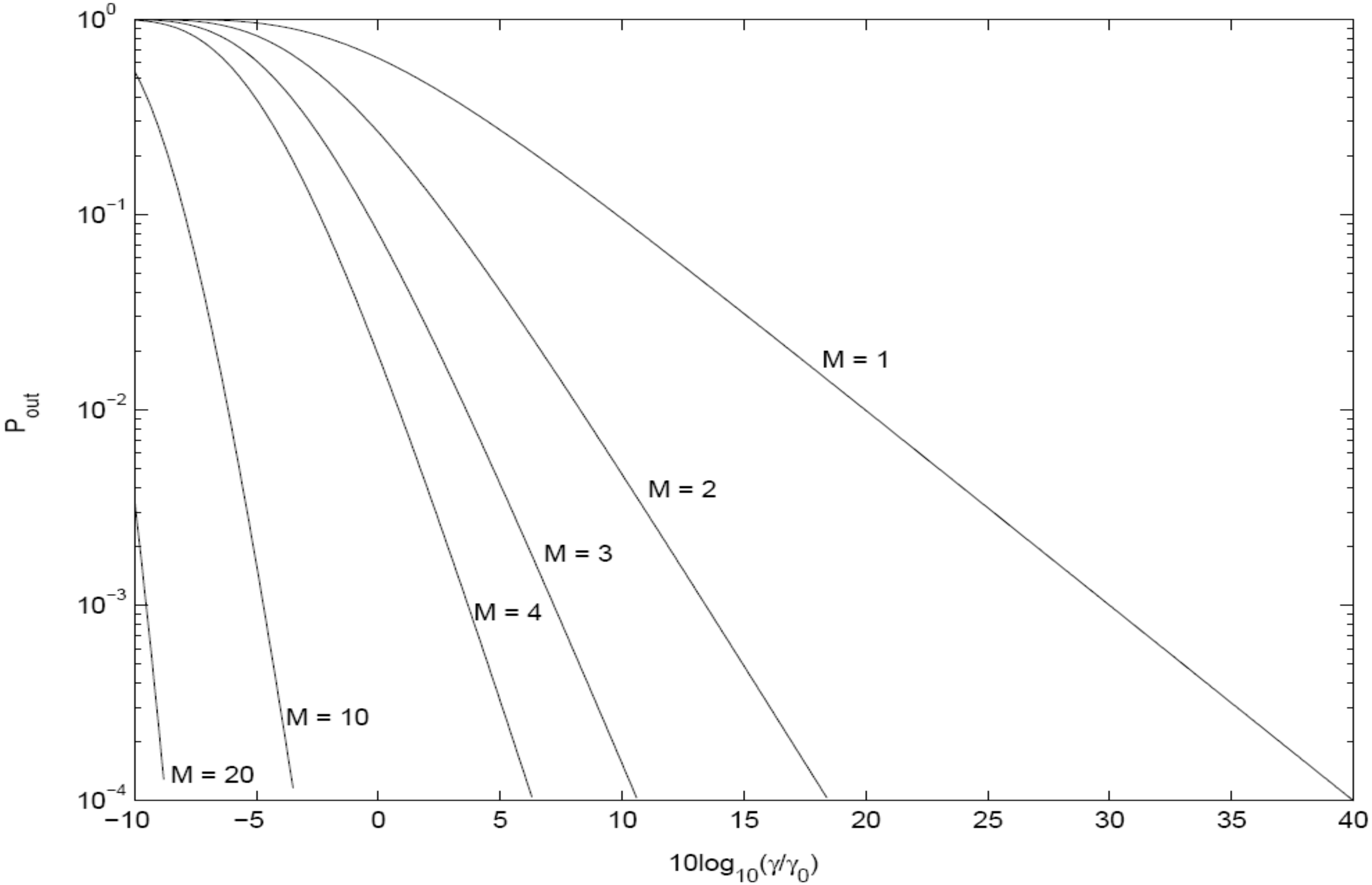
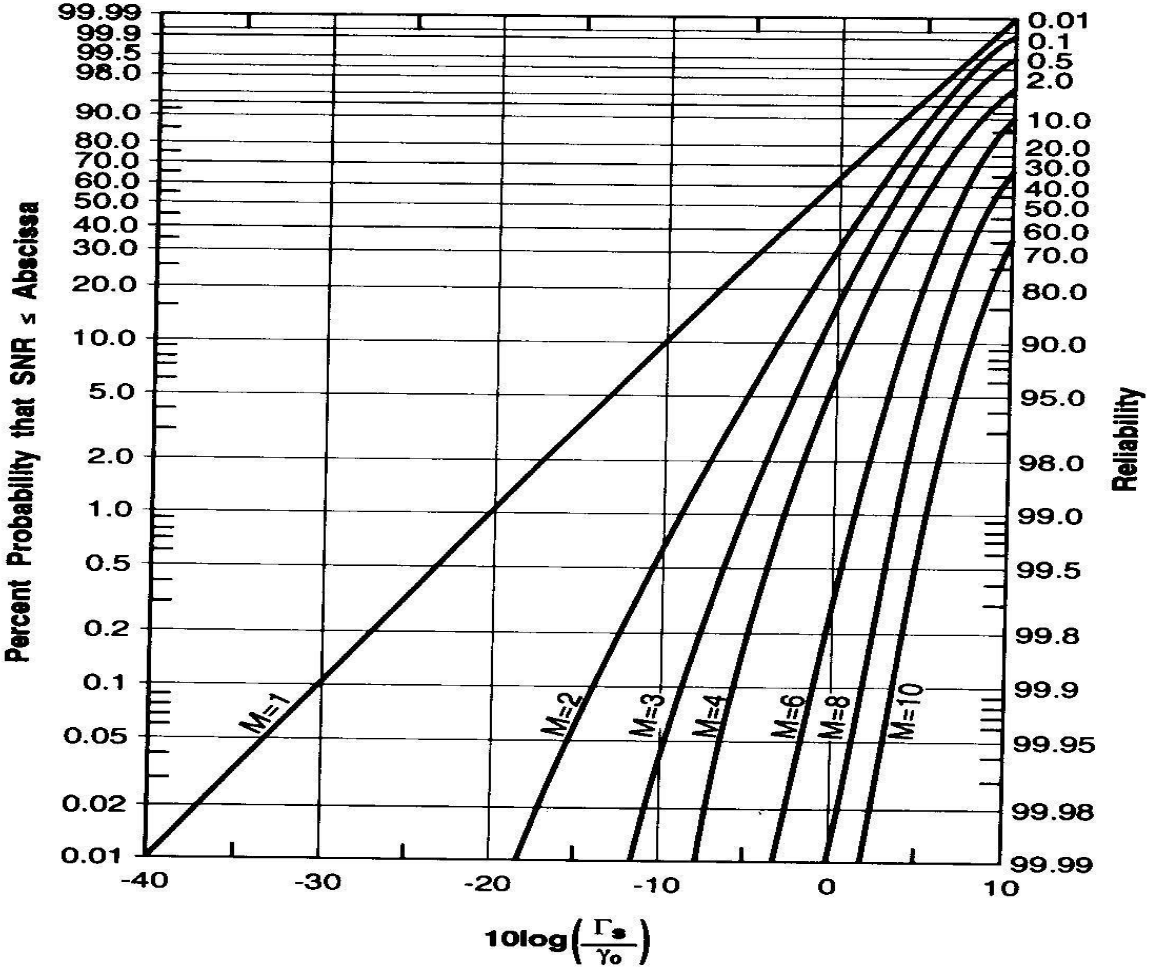


Figure 5.5: P_{out} for MRC with i.i.d Rayleigh fading

Performance of gain combining



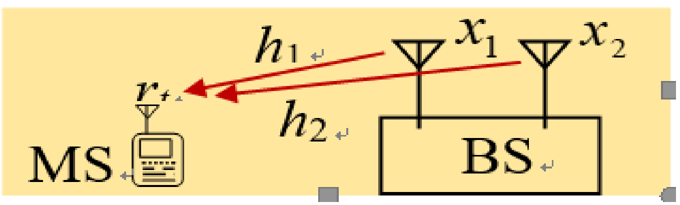
Distribution of the SNR at the output of the equal-gain combiner.

Be careful.
Here is %

Maximum ratio transmission (MRT)

Consider a system with M transmit antennas and one receive antenna. Let h_i and x_i be the path coefficient and signal, respectively, from the ith antenna. The received signal is

$$r = \sum_{i=1}^{M} h_i x_i + \eta \qquad (5.13)$$



We first consider $x_i = d$ for all i, where d is an information symbol with average power P. The received signal then is given by

$$r = \left(\sum_{i=1}^{M} h_i\right) d + \eta \qquad (5.14a)$$

Assumed that all $\{h_i\}$ are independent to each other, so Σh_i in (7.14a) adds non-coherently (since $\{h_i\}$ may cancel each other). Thus the SNR is

$$SNR_{\text{non-coherent}} = \sum_{i=1}^{M} |h_i|^2 P / \sigma^2$$

Note that the total transmitted power is MP, so the transmitted SNR is MP/σ^2. The power gain is

$$\text{power gain} = (1/M) \cdot \sum_{i=1}^{M} |h_i|^2 \qquad (5.14b)$$

Maximum ratio transmission (MRT)

We can do better using maximum ratio transmission (MRT) with

$$x_i = \overline{h}_i d \qquad (5.15a)$$

Substituting (5.15a) into (5.13), we have

$$r = \sum_{i=0}^{M} |h_i|^2 d + \eta$$

Here $\Sigma|h_i|^2$ adds coherently. The received SNR is

$$SNR = \left(\sum_{i=1}^{M} |h_i|^2 \right)^2 P / \sigma^2$$

The total transmitted SNR is $\Sigma|h_i|^2 P/\sigma^2$, so SNR gain is

$$\text{SNR gain} = \sum_{i=1}^{M} |h_i|^2 \qquad (5.15b)$$

This is M times of the non-coherent scheme in (5.14b).

MRT requires to know $\{h_i\}$, which is referred to as channel state information at transmitter (CSIT). This knowledge enables coherent signal combing at the receiver.

Power gain with MRT

SNR after the combiner:	$SNR = \left(\sum_{i=1}^{M} \lvert h_i \rvert^2\right)^2 P / \sigma^2$
SNR at the transmitter:	$\sum_{i=1}^{M} \lvert h_i \rvert^2 P / \sigma^2$
SNR gain:	$\sum_{i=1}^{M} \lvert h_i \rvert^2$

The power gain is the same as MRC.

5.2 Space-time coding

Space-time coding

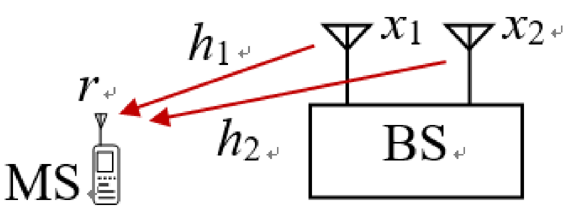
Recall that the power gain for MRT is

$$\sum_{i=1}^{M} |h_i|^2$$

MRT achieves this using CSIT. In practice, accurate CSIT can be a difficult requirement. Is there any advantage in employing multiple antennas at a transmitter without CSIT?

As a starting point, consider a simple repetition scheme. Let $x_1 = x_2 \ldots = x_M = x$ be the signals transmitted from M antennas with power P on each antenna. The received signal is

$$r = \sum_{i=1}^{M} h_i x = hx$$



The total transmitted power for all $\{x_i\}$ is MP. The power gain is

$$\frac{\mathrm{E}\left(\sum_{i=1}^{M} |h_i|^2\right) P}{(MP)} = \frac{1}{M} \cdot \sum_{i=1}^{M} \mathrm{E}\left(|h_i|^2\right)$$

This is M times lower than MRT.

Space-time coding

A more serious problem is that the overall channel coefficient $h=\Sigma h_i$ is Rayleigh distributed. Such distribution is not preferred in many cases.

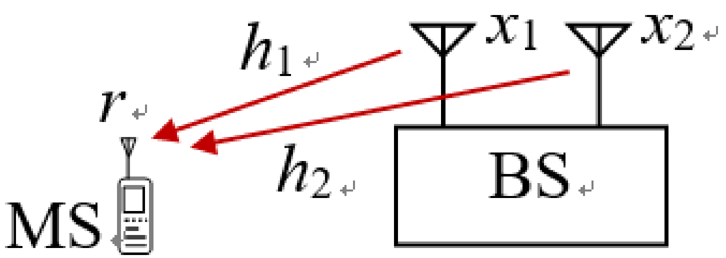
To see the problem, let $x_1=\pm 1$ $h_1 = -h_2$. Even if $|h_1|$ and $|h_2|$ are large (that means h_1 and h_2 are not in deep fades), the received signal can still be zero. This can be a bad situation.

In the following, we will study a space-time coding technique that does not require CSIT, but can increase the diversity (i.e., increasing the free-degree of the power distribution of the received signal).

Space-time block coding

Space-time block coding (STBC) is a technique to achieve diversity with multiple transmit antennas without CSIT. The key is to adopt a "block" coding structure in both spatial and temporal (time) domains. There are other alternatives, such as space-time trellis coding, but we will only discuss STBC below. Our focus is the Alamouti scheme

transmit antenna diversity



Consider M=2. This leads to a 2×1 system (i.e., two transmit antennas and one receive antenna). We introduce a time index t. The received signal is given by

$$r_t = h_1 x_{t,1} + h_2 x_{t,2} + \eta_t \qquad (5.17)$$

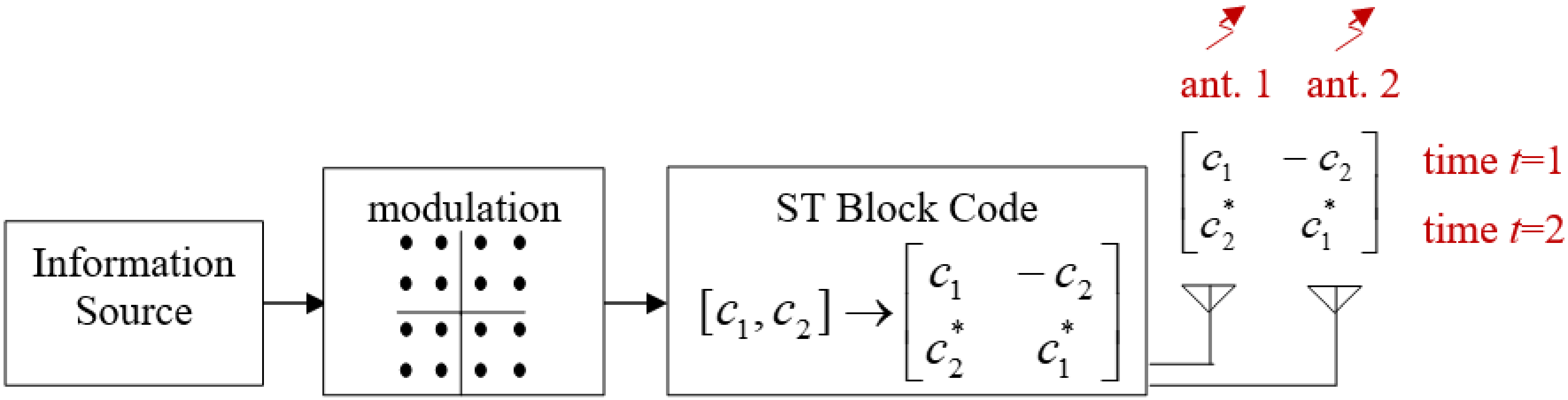
Space-time block coding

In particular, for t=1 and 2, we have

$$r_1 = h_1 x_{1,1} + h_2 x_{1,2} + \eta_1$$
$$r_2 = h_1 x_{2,1} + h_2 x_{2,2} + \eta_2 \qquad (5.18)$$

The Alamouti STBC scheme is defined by the following choice of x.

$$\begin{bmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \end{bmatrix} = \begin{bmatrix} c_1 & -c_2 \\ c_2^* & c_1^* \end{bmatrix} \qquad (5.19)$$



Decoding for the Alamouti scheme

The received signals in (5.18) become:

$$\begin{aligned} r_1 &= h_1 c_1 - h_2 c_2 + \eta_1 \\ r_2 &= h_1 c_2^* + h_2 c_1^* + \eta_2 \end{aligned} \qquad (5.20)$$

In general, $\{c_1, c_2\}$ can be two QAM modulated symbols carrying information. The received signal in (5.20) can be rewritten as

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & -h_2 \\ h_2^* & h_1^* \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2^* \end{bmatrix} \qquad (5.21)$$

Assume that $\{h_1, h_2\}$ are known at the receiver. We now do the following transformation:

$$\begin{bmatrix} \tilde{r}_1 \\ \tilde{r}_2 \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ -h_2^* & h_1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \tilde{\eta}_1 \\ \tilde{\eta}_2^* \end{bmatrix} \qquad (5.22)$$

where $\rho = h_1^* h_1 + h_2^* h_2 = |h_1|^2 + |h_2|^2$ and $\begin{bmatrix} \tilde{\eta}_1 \\ \tilde{\eta}_2^* \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ -h_2^* & h_1 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2^* \end{bmatrix}$

SNR performance

The noise variance and the SNR related to $\{\tilde{r}_1, \tilde{r}_2\}$ is (5.22) are

$$\mathrm{Var}(\tilde{\eta}_1) = \mathrm{Var}(\tilde{\eta}_2) = |h_1|^2 \sigma^2 + |h_2|^2 \sigma^2 = \rho\sigma^2 \qquad (5.23)$$

$$SNR = \frac{\rho^2 P}{\rho\sigma^2} = \left(|h_1|^2 + |h_2|^2\right)\left(P/\sigma^2\right) \qquad (5.24)$$

where we have assumed that the average power of c_i is P. Since each c_i is transmitted twice, $P_{\text{transmitted}} = 2P$ for Alamouti. Therefore we have:

SNR after space-time decoder: $$\sum_{i=1}^{2} |h_i|^2 P/\sigma^2 \qquad (5.25)$$

SNR gain: $$0.5\sum_{i=1}^{2} |h_i|^2 \qquad (5.26)$$

This is compared with MRT in (5.15). MRT is based on the assumption of knowing CSIT, which leads to an advantage because of more power can be allocated to directions with better gain.

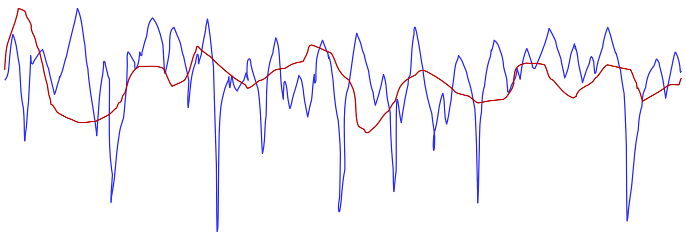
SNR performance

It is also interesting to compare Alamouti with the simple repetition scheme in beginning of 5.2. Both have the same SNR gain. This means that they both cannot provide coherent effect at the receiver. The reason is that they both do not require channel information at the transmitter.

However, repetition transmission results in a Chi–square distribution of free-degree of 2 while Alamouti has results in a Chi–square distribution of free-degree of 4. Thus Alamouti has better diversity.

- What does diversity mean here?
- How dose Almounti achieve diversity?

We should pay attention to ρ in (5-22) for the above questions.



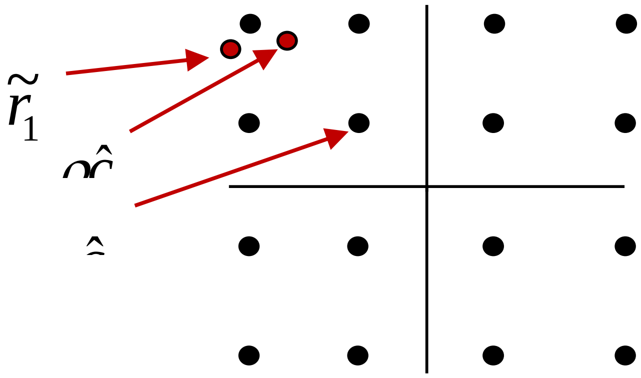
Estimate $\{c_1, c_2\}$

Let C be the set of modulated symbols. Based on (5.22), we have

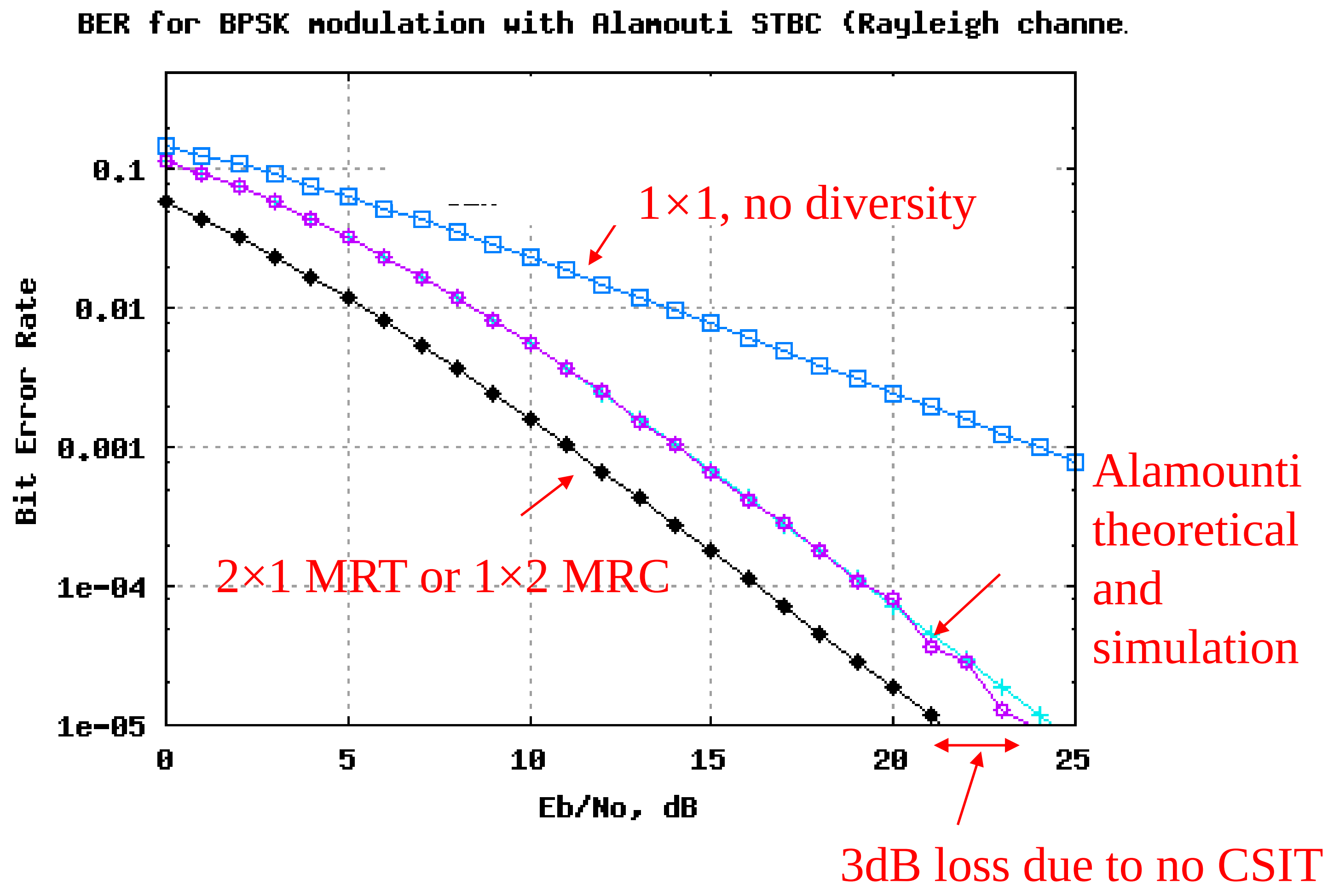
$$\tilde{r}_1 = \rho c_1 + \tilde{\eta}_1$$

The optimal estimation of c_1 is $\hat{c} \in C$ that minimizes $\|\tilde{r}_1 - \rho\hat{c}\|^2$. A similar result applies to c_2. This is the so-called least square principle. We can also use the minimum mean square error principle, for which we will omit details.

Note: The optimal solution is not the signaling point nearest to $\tilde{r}_1$.

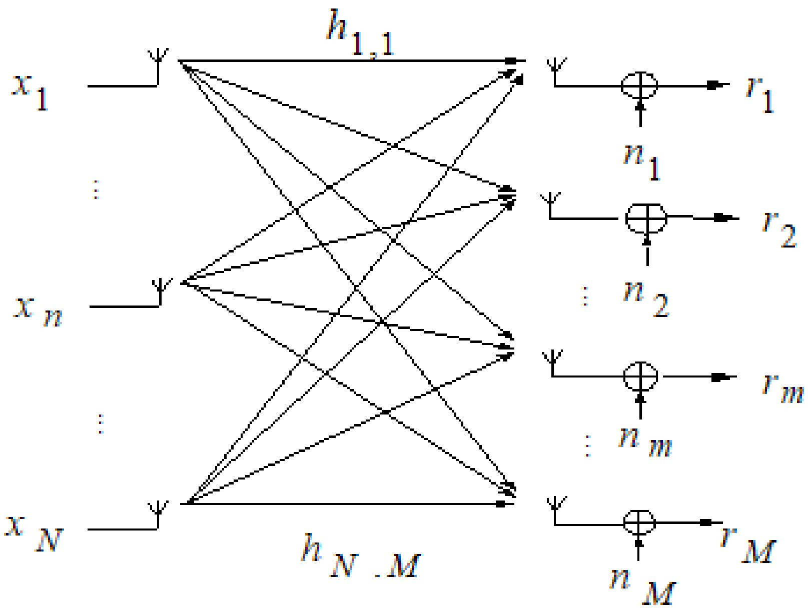


Performance of Alamouti STBC coding



5.3 MIMO

MIMO



A MIMO system can be characterized by a matrix equation:

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\eta}\ .$$

The problem here is that $\boldsymbol{H}$ is full. For example, for 2×2 system, we have

$$r_1 = h_{1,1}x_1 + h_{1,2}x_2 + \eta_1$$
$$r_2 = h_{2,1}x_1 + h_{2,2}x_2 + \eta_2$$

It is difficult to find y_1 and y_2 directly from r_1 and r_2 due to interference.

MIMO

Assume that $\boldsymbol{H}$ is known. We may claim $\boldsymbol{x} \approx \boldsymbol{H}^{-1}\mathbf{y}$. This is referred to as the zero-forcing method. It is not accurate due to the presence of $\boldsymbol{\eta}$.

The singular value decomposition (SVD) technique is a more efficient alternative. Let the SVD of $\boldsymbol{H}$ be

$$\boldsymbol{H} = \boldsymbol{U\Lambda V}^{\mathrm{H}}$$

We use a precoder at the transmitter as

$$\boldsymbol{x} = \boldsymbol{Vc}.$$

where $\boldsymbol{c}$ is the transmitted signal and $\boldsymbol{c}$ represent the actual information. At the receiver, we perform a combing operation as

$$\boldsymbol{z} = \boldsymbol{U}^{\mathrm{H}}\boldsymbol{r}.$$

Clearly,

$$\boldsymbol{z} = \boldsymbol{U}^{\mathrm{H}}\boldsymbol{U\Lambda V}^{\mathrm{H}}\boldsymbol{x} + \boldsymbol{U}^{\mathrm{H}}\boldsymbol{\eta} = \boldsymbol{\Lambda V}^{\mathrm{H}} x + \boldsymbol{\eta}',$$

where $\boldsymbol{\eta}' = \boldsymbol{U}^{\mathrm{H}}\eta$. Noting that $x = Vc$, we have

$$\boldsymbol{z} = \boldsymbol{\Lambda c} + \boldsymbol{\eta}'.$$

We can now detection $\boldsymbol{c}$ in a symbol-by-symbol way.

MIMO

The advantages of the SVD technique are as follows。

(i) It provides good diversity.
(ii) It allows power allocation on different values, which improves transmission efficiency. We will cover the related water-filling technique later after discussing coding principles.

The disadvantages are as follows.

(i) It is necessary to know channel coefficients at the transmitter.
(ii) Eigenvalue decomposition has high computational cost.

Chapter 5 summary

1) Outage probability of switched combining

$$P(\gamma_0) = \Pr(\gamma_{\max} < \gamma_0) = (1 - e^{-\frac{\gamma_0}{\bar{\gamma}}})^M$$

2) Power gains

EGC: $\left(\sum_{i=1}^{M} |h_i|\right)^2 / M$

MRC or MRT: $\sum_{i=1}^{M} |h_i|^2$

Non-coherent: $\sum_{i=1}^{M} |h_i|^2 / M$ (degree of freedom=2)

Alamouti: $0.5\sum_{i=1}^{2} |h_i|^2$ (degree of freedom=4)

3) For IID complex Gaussian $\{\xi_i\}, \sum_{i=1}^{M} |\xi_i|^2$ is chi-square distributed (or χ^2 distributed) with free-degree $2M$.

4) MRC or MRT provides an average power gain of M times compared with the non-coherent transmitting and receiving schemes.

Chapter 5 summary

5) Alamouti coding and decoding

$$\begin{bmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \end{bmatrix} = \begin{bmatrix} c_1 & -c_2 \\ c_2^* & c_1^* \end{bmatrix}$$

$$\begin{bmatrix} \tilde{r}_1 \\ \tilde{r}_2 \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ -h_2^* & h_1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \tilde{\eta}_1 \\ \tilde{\eta}_2^* \end{bmatrix}$$

where $\tilde{r}_1 = \rho c_1 + \tilde{\eta}_1$. The optimal estimate of c_1 is $\hat{c} \in C$ that minimizes $\|\tilde{r}_1 - \rho\hat{c}\|^2$. (Same applies to $\tilde{r}_2$). The power gain of Alamouti is $0.5(|h_1|^2+|h_2|^2)$. This is half of MRT. Its degree of freedom is 4.

6) A MIMO system can be transformed into the following form,

$$\boldsymbol{z} = \boldsymbol{\Lambda c} + \boldsymbol{\eta}',$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix consisting of the eigenvalues of $\boldsymbol{H}$.